The European Union

EDICT OF GOVERNMENT

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EN 1993-4-1

Eurocode 3 - Design of steel structures - Part 4-1: Silos

This European Standard was approved by CEN on 12 June 2006.

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This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the CEN Management Centre has the same status as the official versions.

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Foreword

This European Standard EN 1993-4-1, “Eurocode 3: Design of steel structures – Part 4-1: Silos”, has been prepared by Technical Committee CEN/TC250 « Structural Eurocodes », the Secretariat of which is held by BSI. CEN/TC250 is responsible for all Structural Eurocodes.

This European Standard shall be given the status of a National Standard, either by publication of an identical text or by endorsement, at the latest by August 2007 and conflicting National Standards shall be withdrawn at latest by March 2010.

This Eurocode supersedes ENV 1993-4-1:1999.

According to the CEN-CENELEC Internal Regulations, the National Standard Organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland and United Kingdom.

Background of the Eurocode programme

In 1975, the Commission of the European Community decided on an action programme in the field of construction, based on article 95 of the Treaty. The objective of the programme was the elimination of technical obstacles to trade and the harmonisation of technical specifications.

Within this action programme, the Commission took the initiative to establish a set of harmonised technical rules for the design of construction works which, in a first stage, would serve as an alternative to the national rules in force in the Member States and, ultimately, would replace them.

For fifteen years, the Commission, with the help of a Steering Committee with Representatives of Member States, conducted the development of the Eurocodes programme, which led to the first generation of European codes in the 1980’s.

In 1989, the Commission and the Member States of the EU and EFTA decided, on the basis of an agreement 1) between the Commission and CEN, to transfer the preparation and the publication of the Eurocodes to the CEN through a series of Mandates, in order to provide them with a future status of European Standard (EN). This links de facto the Eurocodes with the provisions of all the Council’s Directives and/or Commission’s Decisions dealing with European standards (e.g. the Council Directive 89/106/EEC on construction products - CPD - and Council Directives 93/37/EEC, 92/50/EEC and 89/440/EEC on public works and services and equivalent EFTA Directives initiated in pursuit of setting up the internal market).

The Structural Eurocode programme comprises the following standards generally consisting of a number of Parts:

<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>EN1990</td>
<td>Eurocode: Basis of structural design</td>
</tr>
<tr>
<td>EN1991</td>
<td>Eurocode 1: Actions on structures</td>
</tr>
<tr>
<td>EN1992</td>
<td>Eurocode 2: Design of concrete structures</td>
</tr>
<tr>
<td>EN1993</td>
<td>Eurocode 3: Design of steel structures</td>
</tr>
<tr>
<td>EN1994</td>
<td>Eurocode 4: Design of composite steel and concrete structures</td>
</tr>
<tr>
<td>EN1995</td>
<td>Eurocode 5: Design of timber structures</td>
</tr>
</tbody>
</table>

1) Agreement between the Commission of the European Communities and the European Committee for Standardisation (CEN) concerning the work on EUROCODES for the design of building and civil engineering works (BC/CEN/03/89).
Eurocode standards recognise the responsibility of regulatory authorities in each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level where these continue to vary from State to State.

**Status and field of application of Eurocodes**

The Member States of the EU and EFTA recognise that EUROCODES serve as reference documents for the following purposes:

- as a basis for specifying contracts for construction works and related engineering services;
- as a framework for drawing up harmonised technical specifications for construction products (ENs and ETAs)

The Eurocodes, as far as they concern the construction works themselves, have a direct relationship with the Interpretative Documents referred to in Article 12 of the CPD, although they are of a different nature from harmonised product standards. Therefore, technical aspects arising from the Eurocodes work need to be adequately considered by CEN Technical Committees and/or EOTA Working Groups working on product standards with a view to achieving full compatibility of these technical specifications with the Eurocodes.

The Eurocode standards provide common structural design rules for everyday use for the design of whole structures and component products of both a traditional and an innovative nature. Unusual forms of construction or design conditions are not specifically covered and additional expert consideration will be required by the designer in such cases.

**National Standards implementing Eurocodes**

The National Standards implementing Eurocodes will comprise the full text of the Eurocode (including any annexes), as published by CEN, which may be preceded by a National title page and National foreword, and may be followed by a National Annex.

The National Annex may only contain information on those parameters which are left open in the Eurocode for national choice, known as Nationally Determined Parameters, to be used for the design of buildings and civil engineering works to be constructed in the country concerned, i.e.:

2) According to Art. 3.3 of the CPD, the essential requirements (ERs) shall be given concrete form in interpretative documents for the creation of the necessary links between the essential requirements and the mandates for harmonised ENs and ETAGs/ETAs.

3) According to Art. 12 of the CPD the interpretative documents shall:
   a) give concrete form to the essential requirements by harmonising the terminology and the technical bases and indicating classes or levels for each requirement where necessary;
   b) indicate methods of correlating these classes or levels of requirement with the technical specifications, e.g. methods of calculation and of proof, technical rules for project design, etc.;
   c) serve as a reference for the establishment of harmonised standards and guidelines for European technical approvals.

The Eurocodes, de facto, play a similar role in the field of the ER 1 and a part of ER 2.
BS EN 1993-4-1:2007
EN 1993-4-1:2007 (E)

- values and/or classes where alternatives are given in the Eurocode,
- values to be used where a symbol only is given in the Eurocode,
- country specific data (geographical, climatic, etc), e.g. snow map,
- the procedure to be used where alternative procedures are given in the Eurocode.

It may also contain:
- decisions on the application of informative annexes,
- references to non-contradictory complementary information to assist the user to apply the Eurocode.

Links between Eurocodes and harmonised technical specifications (ENs and ETAs) for products

There is a need for consistency between the harmonised technical specifications for construction products and the technical rules for works. Furthermore, all the information accompanying the CE Marking of the construction products which refer to Eurocodes should clearly mention which Nationally Determined Parameters have been taken into account.

Additional information specific to EN1993-4-1

EN 1993-4-1 gives design guidance for the structural design of silos.

EN 1993-4-1 gives design rules that supplement the generic rules in the many parts of EN 1993-1.

EN 1993-4-1 is intended for clients, designers, contractors and relevant authorities.

EN 1993-4-1 is intended to be used in conjunction with EN 1990, with EN 1991-4, with the other Parts of EN 1991, with EN 1993-1-6 and EN 1993-4-2, with the other Parts of EN 1993, with EN 1992 and with the other Parts of EN 1994 to EN 1999 relevant to the design of silos. Matters that are already covered in those documents are not repeated.

Numerical values for partial factors and other reliability parameters are recommended as basic values that provide an acceptable level of reliability. They have been selected assuming that an appropriate level of workmanship and quality management applies.

Safety factors for ‘product type’ silos (factory production) can be specified by the appropriate authorities. When applied to ‘product type’ silos, the factors in 2.9 are for guidance purposes only. They are provided to show the likely levels needed to achieve consistent reliability with other designs.

National Annex for EN1993-4-1

This standard gives alternative procedures, values and recommendations for classes with notes indicating where national choices may have to be made. Therefore the National Standard implementing EN 1993-4-1 should have a National Annex containing all Nationally Determined Parameters to be used for the design of buildings and civil engineering works to be constructed in the relevant country.

National choice is allowed in EN 1993-4-1 through:

- 2.2 (i)
- 2.2 (3)

---

4) see Art.3.3 and Art.12 of the CPD, as well as clauses 4.2, 4.3.1, 4.3.2 and 5.2 of ID.1.
1 General

1.1 Scope

(1) Part 4.1 of Eurocode 3 provides principles and application rules for the structural design of steel silos of circular or rectangular plan-form, being free standing or supported.

(2) The provisions given in this Part supplement modify or supersede the equivalent provisions given in EN 1993-1.

(3) This part is concerned only with the requirements for resistance and stability of steel silos. For other requirements (such as operational safety, functional performance, fabrication and erection, quality control, details like man-holes, flanges, filling devices, outlet gates and feeders etc.), see the relevant standards.

(4) Provisions relating to special requirements of seismic design are provided in EN 1998-4, which complements or adapts the provisions of Eurocode 3 specifically for this purpose.

(5) The design of supporting structures for the silo are dealt with in EN 1993-1-1. The supporting structure is deemed to consist of all structural elements beneath the bottom flange of the lowest ring of the silo, see figure 1.1.


(7) Numerical values of the specific actions on steel silos to be taken into account in the design are given in EN 1991-4 Actions in Silos and Tanks.

(8) This Part 4.1 does not cover:
- resistance to fire;
- silos with internal subdivisions and internal structures;
- silos with capacity less than 100 kN (10 tonnes);
- cases where special measures are necessary to limit the consequences of accidents.

(9) Where this standard applies to circular planform silos, the geometric form is restricted to axisymmetric structures, but the actions on them may be unsymmetrical, and their supports may induce forces in the silo that are not axisymmetrical.

1.2 Normative references

This European Standard incorporates, by dated and undated reference, provisions from other standards. These normative references are cited at the appropriate places in the text and the publications are listed hereafter. For dated references, subsequent amendments to, or revisions of, any of these publications apply to the European Standard only when incorporated in it by amendment or revision. For undated references the latest edition of the publication referred to applies.

EN 1090 Execution of steel structures;
EN 1990 Eurocode: Basis of design;
EN 1991 Eurocode 1: Actions on structures;
  Part 1.1 Actions on structures – Densities, self-weight and imposed loads for buildings;
  Part 1.2: Actions on structures – Actions on structures exposed to fire;
  Part 1.3: Actions on structures – Snow loads;
  Part 1.4: Actions on structures – Wind loads;
1.3 Assumptions

(1) In addition to the general assumptions of EN 1990 the following assumptions apply:
   - fabrication and erection complies with EN 1090-2

1.4 Distinction between principles and application rules

(1) See 1.4 in EN 1990.

1.5 Terms and definitions

(1) The terms that are defined in 1.5 in EN 1990 for common use in the Structural Eurocodes and the definitions given in ISO 8930 apply to this Part 4.1 of EN 1993, unless otherwise stated, but for the purposes of this Part 4.1 the following supplementary definitions are given:

1.5.1 shell. A structure formed from a curved thin plate.
1.5.2 **axisymmetric shell.** A shell structure whose geometry is defined by rotation of a meridional line about a central axis.

1.5.3 **box.** A structure formed from an assembly of flat plates into a three-dimensional enclosed form. For the purposes of this Standard, the box has dimensions that are generally comparable in all directions.

1.5.4 **meridional direction.** The tangent to the silo wall in a vertical plane at any point. It varies according to the structural element being considered. Alternatively, it is the vertical or inclined direction on the surface of the structure that a rain drop would take in sliding down the surface.

1.5.5 **circumferential direction.** The horizontal tangent to the silo wall at any point. It varies around the silo, lies in the horizontal plane and is tangential to the silo wall irrespective of whether the silo is circular or rectangular in plan.

1.5.6 **middle surface.** This term is used to refer to both the stress-free middle surface when a shell is in pure bending and the middle plane of a flat plate that forms part of a box.

1.5.7 **separation of stiffeners.** The centre to centre distance between the longitudinal axes of two adjacent parallel stiffeners.

Supplementary to Part 1 of EN 1993 (and Part 4 of EN 1991), for the purposes of this Part 4.1, the following terminology applies, see figure 1.1:

1.5.8 **silo:** A silo is a vessel for storing particulate granular solids. In this Standard, it is assumed to have a vertical form with solids being added by gravity at the top. The term silo includes all forms of particulate solids storage structure, that might otherwise be referred to as a bin, hopper, grain tank or bunker.

1.5.9 **barrel:** The barrel is the vertical walled section of a silo.

1.5.10 **hopper:** A hopper is a converging section towards the bottom of a silo. It is used to channel solids towards a gravity discharge outlet.

1.5.11 **junction:** A junction is the point at which any two or more shell segments, or two or more flat plate elements of a box meet. It can include a stiffener or not: the point of attachment of a ring stiffener to the shell or box may be treated as a junction.

1.5.12 **transition junction:** The transition junction is the junction between the barrel and hopper. The junction can be at the base of the barrel or part way down it.

1.5.13 **skirt:** The skirt is that part of the barrel which lies below the transition junction: it differs from the higher part in that it has no contact with the stored bulk solids.

1.5.14 **strake:** A strake or course is a single layer of steel plates used to form one level of the cylindrical barrel of a silo.

1.5.15 **stringer stiffener:** A stringer stiffener is a local stiffening member that follows the meridian of a shell, representing a generator of the shell of revolution. It is provided to increase the stability, or to assist with the introduction of local loads or to carry axial loads. It is not intended to provide a primary load carrying capacity for bending due to transverse loads.

1.5.16 **rib:** A rib is a local member that provides a primary load carrying path for loads causing bending down the meridian of a shell or flat plate, representing a generator of the shell of revolution.
or a vertical stiffener on a box. It is used to distribute transverse loads on the structure by bending action.

1.5.17 **ring stiffener**: A ring stiffener is a local stiffening member that passes around the circumference of the structure at a given point on the meridian. It is assumed to have no stiffness in the meridional plane of the structure. It is provided to increase the stability or to introduce local loads, not as a primary load-carrying element. In a shell of revolution it is circular, but in rectangular structures it takes the rectangular form of the plan section.

1.5.18 **smeared stiffeners**: Stiffeners are said to be smeared when the properties of the shell wall and the individual stiffeners are treated as a composite section using a width equal to an integer multiple of the separation of the stiffeners. The stiffness properties of a shell wall with smeared stiffeners are orthotropic with eccentric terms leading to coupling between bending and stretching behaviour.

![Figure 1.1: Terminology used in silo structures](image)

**Figure 1.1**: Terminology used in silo structures

1.5.19 **base ring**: A base ring is a structural member that passes around the circumference of the structure at the base and provides means of attachment of the structure to a foundation or other element. It is required to ensure that the assumed boundary conditions are achieved in practice.

1.5.20 **ring girder or ring beam**: A ring girder or ring beam is a circumferential stiffener which has bending stiffness and strength both in the plane of the circular section of a shell or the plan section of a rectangular structure and also normal to that plane. It is a primary load-carrying element, used to distribute local loads into the shell or box structure.

1.5.21 **continuous support**: A continuously supported silo is one in which all positions around the circumference are supported in an identical manner. Minor departures from this condition (e.g. a small opening) need not affect the applicability of the definition.

1.5.22 **discrete support**: A discrete support is a position in which a silo is supported using a local bracket or column, giving a limited number of narrow supports around the silo circumference. Four or six discrete supports are commonly used, but three or more than six are also found.
1.5.23 pyramidal hopper: A pyramidal hopper is used for the hopper section of a rectangular silo, in the form of an inverted pyramid. In this Standard, it is assumed that the geometry is simple, consisting of only four planar elements of trapezoidal shape.

1.6 Symbols used in Part 4.1 of Eurocode 3

The symbols used are based on ISO 3898: 1987.

1.6.1 Roman upper case letters
- \( A \) area of cross-section;
- \( C \) membrane stretching stiffness;
- \( C \) buckling coefficient;
- \( D \) bending flexural rigidity;
- \( E \) Young's modulus;
- \( F \) force;
- \( G \) shear modulus;
- \( H \) height of structure;
- \( I_{\text{II}} \) second moment of area of cross-section;
- \( K \) flexural stiffness of wall panel;
- \( L \) height of shell segment or stiffener;
- \( M \) bending moment;
- \( N \) axial force;
- \( Q \) fabrication tolerance quality of construction of a shell susceptible to buckling;
- \( R_b \) local radius at the crest or trough of a corrugation.

1.6.2 Roman lower case letters
- \( a \) coefficient;
- \( b \) width of plate or stiffener;
- \( d \) crest to crest dimension of a corrugation;
- \( e \) eccentricity of force or stiffener;
- \( f_y \) yield strength of steel;
- \( f_u \) ultimate strength of steel;
- \( h \) separation of flanges of ring girder;
- \( j \) joint efficiency factor for welded lap joints assessed using membrane stresses;
- \( j \) equivalent harmonic of the design stress variation;
- \( \ell \) effective length of shell in linear stress analysis;
- \( \ell \) wavelength of a corrugation in corrugated sheeting;
- \( \ell / \) half wavelength of a potential buckle (height to be considered in calculation);
- \( m \) bending moment per unit width;
- \( m_x \) meridional bending moment per unit circumference;
- \( m_y \) circumferential bending moment per unit height of box;
- \( m_\theta \) circumferential bending moment per unit height of shell;
- \( m_{xy} \) twisting shear moment per unit width of plate;
- \( m_{x,y} \) twisting shear moment per unit width of shell;
- \( n \) membrane stress resultant;
- \( n \) number of discrete supports around silo circumference;
- \( n_x \) meridional membrane stress resultant per unit circumference;
- \( n_y \) circumferential membrane stress resultant per unit height of box;
- \( n_\theta \) circumferential membrane stress resultant per unit height of shell;
- \( n_{xy} \) membrane shear stress resultant per unit width of plate;
- \( n_{x,y} \) membrane shear stress resultant per unit width of shell;
pressure distributed loading;

$p_n$ pressure normal to shell (outward);

$p_s$ meridional surface loading parallel to shell (downward);

$p_a$ circumferential surface loading parallel to shell (anticlockwise in plan);

$q$ transverse force per unit length acting on a tie;

$r$ radial coordinate in a circular plan-form silo;

$r$ radius of shell middle surface;

$s$ circumferential separation of stiffeners;

$t$ wall thickness;

$t_x, t_y$ equivalent wall thickness of corrugated sheet for stretching in the $x$, $y$ directions;

$w$ imperfection amplitude;

$w$ radial deflection;

$x$ local meridional coordinate;

$y$ local circumferential coordinate;

$z$ global axial coordinate;

coordinate along the vertical axis of an axisymmetric silo (shell of revolution).

1.6.3 Greek letters

$\alpha$ elastic buckling imperfection factor (knock-down factor);

$\alpha$ coefficient of thermal expansion;

$\beta$ hopper apex half angle;

$\gamma_p$ partial factor for actions;

$\gamma_M$ partial factor for resistance;

$\delta$ limiting deflection;

$\Delta$ increment;

$\chi$ reduction factor for flexural column buckling;

$\chi$ shell buckling stress reduction factor;

$\lambda$ shell meridional bending half-wavelength;

$\lambda$ relative slenderness of a shell;

$\mu$ wall friction coefficient;

$\nu$ Poisson’s ratio;

$\theta$ circumferential coordinate around shell;

$\sigma$ direct stress;

$\sigma_{bx}$ meridional bending stress;

$\sigma_{by}$ circumferential bending stress in box;

$\sigma_{b\theta}$ circumferential bending stress in curved shell;

$\tau_{bxy}$ twisting shear stress in box;

$\tau_{b\theta x}$ twisting shear stress in curved shell;

$\sigma_{mx}$ meridional membrane stress;

$\sigma_{my}$ circumferential membrane stress in box;

$\sigma_{m\theta}$ circumferential membrane stress in curved shell;

$\tau_{mxy}$ membrane shear stress in box;

$\tau_{m\theta x}$ membrane shear stress in curved shell;

$\sigma_{ox}$ meridional outer surface stress;

$\sigma_{oy}$ circumferential outer surface stress in box;

$\sigma_{o\theta}$ circumferential outer surface stress in curved shell;

$\tau_{oxy}$ outer surface shear stress in box;

$\tau_{o\theta x}$ outer surface shear stress in curved shell;

$\tau$ shear stress;

$\omega$ dimensionless parameter in buckling calculation;
\( \omega \) inclination to vertical of a hopper whose axis is not vertical;
\( \psi \) stress non-uniformity parameter.

### 1.6.4 Subscripts
- \( E \) value of stress or displacement (arising from design actions);
- \( F \) actions;
- \( M \) material;
- \( R \) resistance;
- \( S \) value of stress resultant (arising from design actions);
- \( b \) bending;
- \( c \) cylinder;
- \( cr \) critical buckling value;
- \( d \) design value;
- \( eff \) effective;
- \( h \) hopper;
- \( m \) membrane, midspan;
- \( \min \) minimum allowed value;
- \( n \) normal to the wall;
- \( p \) pressure;
- \( r \) radial;
- \( s \) skirt, support;
- \( s \) surface stress (o... outer surface, i... inner surface)
- \( u \) ultimate;
- \( w \) meridionally parallel to the wall (wall friction);
- \( x \) meridional;
- \( y \) circumferential (box structures), yield;
- \( z \) axial direction;
- \( \theta \) circumferential (shells of revolution).

### 1.7 Sign conventions

#### 1.7.1 Conventions for global silo structure axis system for circular silos

(1) The sign convention given here is for the complete silo structure, and recognises that the silo is not a structural member.

![Coordinate systems for a circular silo](image)
(2) In general, the convention for the global silo structure axis system is in cylindrical coordinates (see figure 1.2) as follows:

**Coordinate system**

<table>
<thead>
<tr>
<th>Coordinate along the central axis of a shell of revolution</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial coordinate</td>
<td>$r$</td>
</tr>
<tr>
<td>Circumferential coordinate</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

(3) The convention for positive directions is:

- Outward direction positive (internal pressure positive, outward displacements positive)
- Tensile stresses positive (except in buckling expressions where compression is positive)

(4) The convention for distributed actions on the silo wall surface is:

- Pressure normal to shell (outward positive) $p_r$
- Meridional surface loading parallel to shell (downward positive) $p_x$
- Circumferential surface loading parallel to shell (anticlockwise positive in plan) $p_\theta$

### 1.7.2 Conventions for global silo structure axis system for rectangular silos

(1) The sign convention given here is for the complete silo structure, and recognises that the silo is not a structural member.

(2) In general, the convention for the global silo structure axis system is in Cartesian coordinates $x$, $y$, $z$, where the vertical direction is taken as $z$, see figure 1.3.

(3) The convention for positive directions is:

- Outward direction positive (internal pressure positive, outward displacements positive)
- Tensile stresses positive (except in buckling expressions where compression is positive)

(4) The convention for distributed actions on the silo wall surface is:

- Pressure normal to box (outward positive) $p_n$
- Meridional surface loading parallel to box surface (downward positive) $p_x$
- Circumferential surface loading in the plane of the box plan cross-section (anticlockwise positive) $p_y$
1.7.3 Conventions for structural element axes in both circular and rectangular silos

(1) The convention for structural elements attached to the silo wall (see figures 1.4 and 1.5) is different for meridional and circumferential members.

(2) The convention for meridional straight structural elements (see figure 1.4a) attached to the silo wall (shells and boxes) is:

- Meridional coordinate for barrel, hopper and roof attachment
- Strong bending axis (parallel to flanges: axis for meridional bending)
- Weak bending axis (perpendicular to flanges)

**NOTE:** A meridional stiffener bending in a manner that is compatible with meridional bending \( \theta_x \) in the cylinder bends about the \( y \) axis of the stiffener.
Figure 1.4: Local coordinate systems for meridional stiffeners on a shell or box

Figure 1.5: Local coordinate systems for circumferential stiffeners on a shell or box

(3) The convention for circumferential curved structural elements (see figure 1.5a) attached to a shell wall is:

- Circumferential coordinate axis (curved) $\theta$
- Radial axis (axis for bending in the vertical plane) $r$
- Vertical axis (axis for circumferential bending) $z$

**NOTE:** A circumferential stiffener or ring is subject to bending about its vertical axis $z$ when the bending is compatible with circumferential bending in the cylinder ($m_\theta$). It is subject to bending moments about its radial axis $r$ when either acting as a ring girder, or when subject to radial forces acting at a point eccentric to the ring centroid.
(4) The convention for circumferential straight structural elements attached to a box is:

Circumferential axis \( x \)

Horizontal axis \( y \)

Vertical axis \( z \)

**NOTE:** A circumferential straight stiffener on a box is subject to bending about its vertical axis \( z \) when the bending is out of the plane of the box wall, which is the normal condition.

### 1.7.4 Conventions for stress resultants for circular silos and rectangular silos

(1) The convention used for subscripts indicating membrane forces is:

"The subscript derives from the direction in which direct stress is induced by the force"

**Membrane stress resultants:**

\[ n_x \] meridional membrane stress resultant

\[ n_\theta \] circumferential membrane stress resultant in shells

\[ n_y \] circumferential membrane stress resultant in rectangular boxes

\[ n_{xy} \] or \[ n_{x\theta} \] membrane shear stress resultant

**Membrane stresses:**

\[ \sigma_{mx} \] meridional membrane stress

\[ \sigma_{m\theta} \] circumferential membrane stress in shells

\[ \sigma_{my} \] circumferential membrane stress in rectangular boxes

\[ \tau_{mxy} \] or \[ \tau_{m\theta} \] membrane shear stress

(2) The convention used for subscripts indicating moments is:

"The subscript derives from the direction in which direct stress is induced by the moment"

**NOTE:** This plate and shell convention differs from that for beams and columns as used in Eurocode 3 Parts 1.1 and 1.3. Care must be exercised when using Parts 1.1 and 1.3 in conjunction with these rules.

**Bending stress resultants:**

\[ m_x \] meridional bending moment per unit width

\[ m_\theta \] circumferential bending moment per unit width in shells

\[ m_y \] circumferential bending stress resultant in rectangular boxes

\[ m_{xy} \] or \[ m_{x\theta} \] twisting shear moment per unit width

**Bending stresses:**

\[ \sigma_{bx} \] meridional bending stress

\[ \sigma_{b\theta} \] circumferential bending stress in shells

\[ \sigma_{by} \] circumferential bending stress in rectangular boxes

\[ \tau_{bxy} \] or \[ \tau_{bx\theta} \] twisting shear stress
Inner and outer surface stresses:

\[ \sigma_{\text{inner}}, \sigma_{\text{outer}} \]  meridional inner, outer surface stress for boxes and shells

\[ \tau_{\text{inner}}, \tau_{\text{outer}} \]  inner, outer surface shear stress in shells

\[ \sigma_{\text{circ}}, \sigma_{\text{circ}} \]  circumferential inner, outer surface stress in shells

\[ \tau_{\text{circ}}, \tau_{\text{circ}} \]  inner, outer surface shear stress in shells

\[ \sigma_{\text{circ}}, \sigma_{\text{circ}} \]  circumferential inner, outer surface stress in rectangular boxes

\[ \tau_{\text{circ}}, \tau_{\text{circ}} \]  inner, outer surface shear stress in rectangular boxes

---

**Figure 1.6: Stress resultants in the silo wall (shells and boxes)**

### 1.8 Units

(1) S.I. units shall be used in accordance with ISO 1000.

(2) For calculations, the following consistent units are recommended:

- dimensions and thicknesses: \( m \) mm
- unit weight: \( \text{kN/m}^3 \) N/mm\(^3\)
- forces and loads: \( \text{kN} \) N
- line forces and line loads: \( \text{kN/m} \) N/mm
- pressures and area distributed actions: \( \text{kPa} \) MPa
- unit mass: \( \text{kg/m}^3 \) kg/mm\(^3\)
- acceleration: \( \text{km/s}^2 \) m/s\(^2\)
- membrane stress resultants: \( \text{kN/m} \) N/mm
- bending stress resultants: \( \text{kNm/m} \) N/mm/mm
- stresses and elastic moduli: \( \text{kPa} \) MPa (=N/mm\(^2\))
2 Basis of design

2.1 Requirements

(1) A silo shall be designed, constructed and maintained to meet the requirements of section 2 of EN 1990 as supplemented by the following.

(2) The silo structure should include all shell and plated sections of the structure, including stiffeners, ribs, rings and attachments.

(3) The supporting structure should not be treated as part of the silo structure. The boundary between the silo and its supports should be taken as indicated in figure 1.1. Similarly, other structures supported by the silo should be treated as beginning where the silo wall or attachment ends.

(4) Silos should be designed to be damage-tolerant where appropriate, considering the use of the silo.

(5) Particular requirements for special applications may be agreed between the designer, the client and the relevant authority.

2.2 Reliability differentiation

(1) For reliability differentiation, see EN 1990.

NOTE: The national annex may define consequence classes for silos as a function of the location, type of infill and loading, the structural type, size and type of operation.

(2) Different levels of rigour should be used in the design of silo structures, depending on the consequence class chosen, the structural arrangement and the susceptibility to different failure modes.

(3) For this standard, 3 consequence classes are used, with requirements which produce designs with essentially equal risk in the design assessment and considering the expense and procedures necessary to reduce the risk of failure for different structures: Consequence Classes 1, 2 and 3.

NOTE 1: The national annex may provide information on the consequence classes. Table 2.1 gives an example for the classification of two parameters, the size and the type of operation into consequence classes when all other parameters result in medium consequences, see EN 1990, B.3.1.
### Table 2.1: Consequence classes depending on size and operation

<table>
<thead>
<tr>
<th>Consequence Class</th>
<th>Design situations</th>
</tr>
</thead>
</table>
| Consequence Class 3 | Ground supported silos or silos supported on a complete skirt extending to the ground with capacity in excess of $W_{3a}$ tonnes  
|                     | Discretely supported silos with capacity in excess of $W_{3b}$ tonnes  
|                     | Silos with capacity in excess of $W_{3c}$ tonnes in which any of the following design situations occur:  
|                     | a) eccentric discharge  
|                     | b) local patch loading  
|                     | c) unsymmetrical filling  
| Consequence Class 2 | All silos covered by this Standard and not placed in another class  
| Consequence Class 1 | Silos with capacity between $W_{1a}$ tonnes† and $W_{1b}$ tonnes  

† Silos with capacity less than $W_{1a}$ tonnes are not covered by this standard.

The recommended values for class boundaries are as follows:

<table>
<thead>
<tr>
<th>Class boundary</th>
<th>Recommended value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{3a}$</td>
<td>5000</td>
</tr>
<tr>
<td>$W_{3b}$</td>
<td>1000</td>
</tr>
<tr>
<td>$W_{3c}$</td>
<td>200</td>
</tr>
<tr>
<td>$W_{1b}$</td>
<td>100</td>
</tr>
<tr>
<td>$W_{1a}$</td>
<td>10</td>
</tr>
</tbody>
</table>

**NOTE 2:** For the classification into action assessment classes, see EN 1991-4.

(4) A higher Consequence Class may always be adopted than that required.

(5) The choice of relevant Consequence Class shall be agreed between the designer, the client and the relevant authority.

(6) Consequence Class 3 should be used for local patch loading, which refers to a stored solids loading case causing a patch load which extends round less than half the circumference of the silo, as defined in EN 1991-4.

(7) For Consequence Class 1, simplified provisions may be adopted.

**NOTE:** Appropriate provisions for silos in Consequence Class 1 are set out in Annex A.

### 2.3 Limit states

(1) The limit states defined in EN 1993-1-6 should be adopted for this Part.
2.4 Actions and environmental effects

2.4.1 General

(1) The general requirements set out in section 4 of EN 1990 shall be satisfied.

2.4.2 Wind action

(1) For specifications of wind actions not set down in EN 1991-1-4 for the design of silos in isolation and in groups, appropriate additional information should be agreed.

(2) Because these large light structures are sensitive to the detailed wind pressure distribution on the wall, both with respect to the buckling resistance when empty and the holding down details required at the foundation, additional information may be used to augment the basic wind data provided in EN 1991-1-4 for the specific needs of individual constructions.

NOTE: Appropriate additional information on wind pressure distributions is set out in Annex C.

2.4.3 Combination of solids pressures with other actions

(1) The partial factors on actions in silos set out in 2.9.2 shall be used.

2.5 Material properties

(1) The general requirements for material properties set out in EN 1993-1-1 should be followed.

(2) The specific properties of materials for silos given in section 3 of this Part should be used.

2.6 Geometrical data

(1) The provisions concerning geometrical data given in section 6 of EN 1990 shall be followed.

(2) The additional information specific to shell structures given in EN 1993-1-6 should also be applied.

(3) The shell plate thickness should be taken as the nominal thickness. In the case of hot-dipped metal coated steel sheet conforming with EN 10149, the nominal thickness should be taken as the nominal core thickness, obtained as the nominal external thickness less the total thickness of zinc coating on both surfaces.

(4) The effects of corrosion and abrasion on the thickness of silo wall plates should be included in the design, in accordance with 4.1.4.

2.7 Modelling of the silo for determining action effects

(1) The general requirements set out in section 7 of EN 1990 shall be followed.

(2) The specific requirements for structural analysis in relation to serviceability, set out in sections 4 to 9 of this Part for each structural segment, should be followed.

(3) The specific requirements for structural analysis in relation to ultimate limit states, set out in sections 4 to 9 of this Part and in more detail in EN 1993-1-6 and EN 1993-1-7, should be followed.

2.8 Design assisted by testing

(1) The general requirements set out in Annex D of EN 1990 should be followed.
(2) For 'product type' silos (factory production), which are subject to full scale testing, 'deemed-to-satisfy' criteria may be adopted for design purposes.

2.9 Action effects for limit state verifications

2.9.1 General

(1) The general requirements set out in section 9 of EN 1990 shall be satisfied.

2.9.2 Partial factors for ultimate limit states

2.9.2.1 Partial factors for actions on silos

(1) For persistent, transient and accidental design situations, the partial factors $\gamma$ shall be taken from EN 1990 and EN 1991-4.

(2) Partial factors for ‘product type’ silos (factory production) may be specified by the appropriate authorities.

NOTE: When applied to ‘product type’ silos, the factors in (1) are for guidance purposes only. They are provided to show the likely levels needed to achieve consistent reliability with other designs.

2.9.2.2 Partial factors for resistances

(1) Where structural properties are determined by testing, the requirements and procedures of EN 1990 should be adopted.

(2) Fatigue verifications should satisfy section 9 of EN 1993-1-6.

(3) The partial factors $\gamma_M$ for different limit states shall be taken from table 2.2.

<table>
<thead>
<tr>
<th>Resistance to failure mode</th>
<th>Relevant $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance of welded or bolted shell wall to</td>
<td>$\gamma_M0$</td>
</tr>
<tr>
<td>plastic limit state</td>
<td></td>
</tr>
<tr>
<td>Resistance of shell wall to stability</td>
<td>$\gamma_M1$</td>
</tr>
<tr>
<td>Resistance of welded or bolted shell wall to</td>
<td>$\gamma_M2$</td>
</tr>
<tr>
<td>rupture</td>
<td></td>
</tr>
<tr>
<td>Resistance of shell wall to cyclic plasticity</td>
<td>$\gamma_M4$</td>
</tr>
<tr>
<td>Resistance of connections</td>
<td>$\gamma_M5$</td>
</tr>
<tr>
<td>Resistance of shell wall to fatigue</td>
<td>$\gamma_M6$</td>
</tr>
</tbody>
</table>

NOTE: Partial factors $\gamma_M$ for silos may be defined in the National Annex. For values of $\gamma_M5$, further information may be found in EN 1993-1-8. For values of $\gamma_M0$, further information may be found in EN 1993-1-9. The following numerical values are recommended for silos:

<table>
<thead>
<tr>
<th>$\gamma_M0$ = 1.00</th>
<th>$\gamma_M1$ = 1.10</th>
<th>$\gamma_M2$ = 1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_M4$ = 1.00</td>
<td>$\gamma_M5$ = 1.25</td>
<td>$\gamma_M6$ = 1.10</td>
</tr>
</tbody>
</table>

For further differentiation, see 2.2(1) and 2.2(3)
2.9.3 Serviceability limit states

(1) Where simplified compliance rules are given in the relevant provisions dealing with serviceability limit states, detailed calculations using combinations of actions need not be carried out.

2.10 Durability

(1) The general requirements set out in 2.6 of EN 1990 should be followed.

2.11 Fire resistance

(1) The provisions set out in EN 1993-1-2 for fire resistance should be met.
3 Properties of materials

3.1 General

(1) All steels used for silos should be suitable for welding to permit later modifications when necessary.

(2) All steels used for silos of circular planform should be suitable for cold forming into curved sheets or curved members.

(3) The material properties given in this section, see Table 3.1 in EN 1993-1-1 and Table 3.1b in EN 1993-1-3, should be treated as nominal values to be adopted as characteristic values in design calculations.

(4) Other material properties are given in the relevant Reference Standards defined in EN 1993-1-1.

(5) Where the silo may be filled with hot solids, the values of the material properties should be appropriately reduced to values corresponding to the maximum temperatures to be encountered.

(6) Where the temperature exceeds 100°C, the material properties should be obtained from EN 13084-7.

3.2 Structural steels

(1) The methods for design by calculation given in this Part 4.1 of EN 1993 may be used for structural steels as defined in EN 1993-1-1, which conform with the European Standards and International Standards listed in table 3.1.

(2) The mechanical properties of structural steels, according to EN 10025 or EN 10149 should be obtained from EN 1993-1-1, EN 1993-1-3 and EN 1993-1-4.

(3) Corrosion and abrasion allowances are given in section 4 of this Part 4.1.

(4) It should be assumed that the properties of steel in tension are the same as those in compression.

(5) For the steels covered by this Part 4.1 of EN 1993, the design value of the modulus of elasticity should be taken as $E = 210,000$ MPa and Poisson’s ratio as $\nu = 0.3$.

3.3 Stainless steels

(1) The mechanical properties of stainless steels should be obtained from EN 1993-1-4.

(2) Guidance for the selection of stainless steels in view of corrosion and abrasion actions of stored solids may be obtained from appropriate sources.

(3) Where the design involves a buckling calculation, appropriate reduced properties should be used (see EN 1993-1-6).

3.4 Special alloy steels

(1) For non-standardised alloy steels, appropriate values of relevant mechanical properties should be defined.

NOTE: The National Annex may give information on appropriate values.
(2) Guidance for the selection of non-standardised alloy steels with respect to the corrosion and abrasion actions of stored solids should be obtained from appropriate sources.

(3) Where the design involves a buckling calculation, appropriate reduced properties should be used (see EN 1993-1-6).

3.5 Toughness requirements

(1) The toughness requirements for the steels should be determined according to EN 1993-1-10.
4 Basis for structural analysis

4.1 Ultimate limit states

4.1.1 Basis

(1) Steel structures and components should be so proportioned that the basic design requirements given in section 2 are satisfied.

4.1.2 Required checks

(1) For every relevant limit state, the design shall satisfy the condition:

\[ S_d < R_d \]  

... (4.1)

where \( S_d \) and \( R_d \) represent any appropriate parameter.

4.1.3 Fatigue and cyclic plasticity - low cycle fatigue

(1) Parts of the structure subject to severe local bending should be checked against the fatigue and cyclic plasticity limit states using the procedures of EN 1993-1-6 and EN 1993-1-7 as appropriate.

(2) Silos in Consequence Class I need not be checked for fatigue or cyclic plasticity.

4.1.4 Allowance for corrosion and abrasion

(1) The effects of abrasion of the stored solid on the walls of the container over the life of the structure should be included in determining the effective thickness of the wall for analysis.

(2) Where no specific information is available, the wall should be assumed to lose an amount \( \Delta t_a \) of its thickness due to abrasion at all points on contact with moving solid.

**NOTE:** The National Annex may choose the value of \( \Delta t_a \). The value \( \Delta t_a = 2\text{mm} \) is recommended.

(3) The effects of corrosion of the wall in contact with the stored solid over the life of the structure should be included in determining the effective thickness of the wall for analysis.

(4) Specific values for corrosion and abrasion losses, appropriate to the intended use, should be agreed between the designer, the client and the relevant authority, taking account of the intended use and the nature of the solids to be stored.

**NOTE 1:** The National Annex may choose appropriate values for corrosion and abrasion losses for particular solids in frictional contact with defined silo wall materials, recognising the mode of solids flow defined in EN 1991-4.

**NOTE 2:** To ensure that the design assumptions are met in service, appropriate inspection measures have to be instituted.

4.1.5 Allowance for temperature effects

(1) Where hot solids are stored in the silo, the effects of differential temperature between parts of the structure in contact with hot material and those that have cooled should be included in determining the stress distribution in the wall.
4.2 **Analysis of the structure of a shell silo**

4.2.1 **Modelling of the structural shell**

(1) The modelling of the structural shell should follow the requirements of EN 1993-1-6. They may be deemed to be satisfied by the following provisions.

(2) The modelling of the structural shell should include all stiffeners, large openings, and attachments.

(3) The design should ensure that the assumed boundary conditions are satisfied.

4.2.2 **Methods of analysis**

4.2.2.1 **General**

(1) The analysis of the silo shell should be carried out according to the requirements of EN 1993-1-6.

(2) A higher class of analysis may always be used than that defined for the Consequence Class.

4.2.2.2 **Consequence Class 3**

(1) For silos in Consequence Class 3 (see 2.3), the internal forces and moments should be determined using a validated numerical analysis (finite element shell analysis) (as defined in EN 1993-1-6). Plastic collapse strengths under primary stress states may be used in relation to the plastic limit state as defined in EN 1993-1-6.

4.2.2.3 **Consequence Class 2**

(1) For silos in Consequence Class 2 under conditions of axisymmetric actions and support, one of two alternative analyses may be used:

a) Membrane theory may be used to determine the primary stresses. Bending theory elastic expressions may be used to describe all local bending effects.

b) A validated numerical analysis may be used (e.g. finite element shell analysis) (as defined in EN 1993-1-6).

(2) Where the design loading from stored solids cannot be treated as axisymmetric, a validated numerical analysis should be used.

(3) Notwithstanding paragraph (2), where the loading varies smoothly around the shell causing global bending only (i.e. in the form of harmonic 1), membrane theory may be used to determine the primary stresses.

(4) For analyses of actions due to wind loading and/or foundation settlement and/or smoothly varying patch loads (see EN 1991-4 for thin walled silos), semi-membrane theory or membrane theory may be used.

(5) Where membrane theory is used to find the primary stresses in the shell:

a) Discrete rings attached to an isotropic cylindrical silo shell under internal pressure may be deemed to have an effective area which includes a length of shell above and below the ring of $0.78\sqrt{rt}$ except where the ring is at a transition junction.

b) The effect of local bending stresses at discontinuities in the shell surface and supports should be evaluated separately.
(6) Where an isotropic shell wall is discretely stiffened by vertical stiffeners, the stresses in the stiffeners and the shell wall may be calculated by treating the stiffeners as smeared on the shell wall, provided the spacing of the stiffeners is no wider than \( n_{ss} \sqrt{rt} \).

**NOTE:** The National Annex may choose the value of \( n_{ss} \). The value \( n_{ss} = 5 \) is recommended.

(7) Where smeared stiffeners are used, the stress in the stiffener should be determined making proper allowance for compatibility between the stiffener and the wall and including the effect of the wall membrane stress in the orthogonal direction.

(8) Where a ring girder is used above discrete supports, membrane theory may be used to determine the primary stresses, but the requirements of 5.4 and 8.1.4 concerning the evaluation of additional non-axisymmetric primary stresses should be followed.

(9) Where a ring girder is used above discrete supports, compatibility of the deformations between the ring and adjacent shell segments should be considered, see Figure 4.1. Particular attention should be paid to compatibility of the axial deformations, as the induced stresses penetrate far up the shell. Where such a ring girder is used, the eccentricity of the ring girder centroid and shear centre relative to the shell wall and the support centreline should be considered, see 8.1.4 and 8.2.3.
Cylindrical shell

Axisymmetric wall loading and bottom pressures

Uniform support to cylinder from ring girder

Ring girder (various cross-section geometries)

Uniform loading of ring girder by cylinder

Discrete local supports

a) Traditional design model for column-supported silos

Shell wall

In-plane vertical deflections

Ring girder deflected shape

Discrete support

Discrete support

b) Deformation requirement on cylinder imposed by compatibility with beam deformation

Figure 4.1: Axial deformation compatibility between ring girder and shell

(10) Where the silo is subject to any form of unsymmetrical bulk solids loading (patch loads, eccentric discharge, unsymmetrical filling etc.), the structural model should be designed to capture the membrane shear transmission within the silo wall and between the wall and rings.

NOTE: The shear transmission between parts of the wall and rings has special importance in construction using bolts or other discrete connectors (e.g. between the wall and hopper, between different strakes of the barrel).

(11) Where a ring girder is used to redistribute silo wall forces into discrete supports, and where bolts or discrete connectors are used to join the structural elements, the shear transmission between the parts of the ring due to shell bending and ring girder bending phenomena should be determined.

(12) Except where a rational analysis is used and there is clear evidence that the solid against the wall is not in motion during discharge, the stiffness of the bulk solid in resisting wall deformations or in increasing the buckling resistance of the structure should not be considered.
4.2.2.4 Consequence Class 1

(1) For silos in Consequence Class 1, membrane theory may be used to determine the primary stresses, with factors and simplified expressions to describe local bending effects and unsymmetrical actions.

4.2.3 Geometric imperfections

(1) Geometric imperfections in the shell should satisfy the limitations defined in EN 1993-1-6.

(2) For silos in Consequence Classes 2 and 3, the geometric imperfections should be measured following construction to ensure that the assumed fabrication tolerance quality has been achieved.

(3) Geometric imperfections in the shell need not be explicitly included in determining the internal forces and moments, except where a GNIA or GMNIA analysis is used, as defined in EN 1993-1-6.

4.3 Analysis of the box structure of a rectangular silo

4.3.1 Modelling of the structural box

(1) The modelling of the structural box should follow the requirements of EN 1993-1-7, but they may be deemed to be satisfied by the following provisions.

(2) The modelling of the structural box should include all stiffeners, large openings, and attachments.

(3) The design should ensure that the assumed boundary conditions are satisfied.

(4) The joints between segments of the box should satisfy the modelling assumptions for strength and stiffness.

(5) Each panel of the box may be treated as an individual plate segment provided that both:
   a) the forces and moments introduced into each panel by its neighbours are included;
   b) the flexural stiffness of adjacent panels is included.

(6) Where an isotropic plate wall panel is discretely stiffened with horizontal stiffeners, the stresses in the stiffeners and the box wall may be calculated by treating the stiffeners as smeared on the wall to produce an orthotropic plate, provided that the spacing of the stiffeners is no wider than \( n_e \times t \).

\[ \text{NOTE: } \text{The National Annex may choose the value of } n_e. \text{ The value } n_e = 40 \text{ is recommended.} \]

(7) Where smeared stiffeners are used, the stress in the stiffener should be determined making proper allowance for the eccentricity of the stiffener from the wall plate, and for the wall stress in the direction orthogonal to the axis of the stiffener.

(8) The effective width of plate on each side of a stiffener should be taken as not greater than \( n_{ew} \times t \), where \( t \) is the local plate thickness.

\[ \text{NOTE: } \text{The National Annex may choose the value of } n_{ew}. \text{ The value } n_{ew} = 15 \text{ is recommended.} \]

4.3.2 Geometric imperfections

(1) Geometric imperfections in the box should satisfy the limitations defined in EN 1993-1-7.
(2) Geometric imperfections in the box need not be explicitly included in determining the internal forces and moments.

4.3.3 Methods of analysis

(1) The internal forces in the plate segments of the box wall may be determined using either:
   
   a) static equilibrium for membrane forces and beam theory for bending;
   
   b) an analysis based on linear plate bending and stretching theory;
   
   c) an analysis based on nonlinear plate bending and stretching theory.

(2) For silos in Consequence Class 1, method (a) in (1) may be used.

(3) Where the design loading condition is symmetric relative to each plate segment and the silo is in Consequence Class 2, method (a) in (1) may be used.

(4) Where the design loading condition is not symmetric and the silo is in Consequence Class 2, either method (b) or method (c) in (1) should be used.

(5) For silos in Consequence Class 3 (see 2.2), the internal forces and moments should be determined using either method (b) or method (c) in (1) (as defined in EN 1993-1-7).

4.4 Equivalent orthotropic properties of corrugated sheeting

(1) Where corrugated sheeting is used as part of the silo structure, the analysis may be carried out treating the sheeting as an equivalent uniform orthotropic wall.

(2) The following properties may be used in a stress analysis and in a buckling analysis of the structure, provided that the corrugation profile has either an arc-and-tangent or a sinusoidal shape. Where other corrugation profiles are used, the corresponding properties should be calculated from first principles.

![Corrugation profile and geometric parameters](image)

**Figure 4.2: Corrugation profile and geometric parameters**

(3) The properties of the corrugated sheeting should be defined in terms of an $x$, $y$ coordinate system in which the $y$ axis runs parallel to the corrugations (straight lines on the surface) whilst $x$ runs normal to the corrugations (troughs and peaks). The corrugation should be defined in terms of the following parameters, irrespective of the actual corrugation profile, see figure 4.2:

where:

- $d$ is the crest to crest dimension;
- $\ell$ is the wavelength of the corrugation;
- $R_\phi$ is the local radius at the crest or trough.
(4) All properties may be treated as one-dimensional, giving no Poisson effects between different directions.

(5) The equivalent membrane properties (stretching stiffnesses) may be taken as:

\[ C_x = E t_x = E \frac{2t^3}{3d^2} \]  \hspace{1cm} \text{(4.2)}

\[ C_y = E t_y = E t \left( 1 + \frac{\pi^2 d^2}{4l^2} \right) \]  \hspace{1cm} \text{(4.3)}

\[ C_{xy} = G t_{xy} = \frac{Gt}{1 + \frac{\pi^2 d^2}{4l^2}} \]  \hspace{1cm} \text{(4.4)}

where:
- \( t_x \) is the equivalent thickness for smeared membrane forces normal to the corrugations;
- \( t_y \) is the equivalent thickness for smeared membrane forces parallel to the corrugations;
- \( t_{xy} \) is the equivalent thickness for smeared membrane shear forces.

(6) The equivalent bending properties (flexural stiffnesses) are defined in terms of the flexural rigidity for moments causing bending in that direction, and may be taken as:

\[ D_x = EI_x \text{ per unit width} = \frac{Et^3}{12(1-\nu^2)} \left( 1 + \frac{\pi^2 d^2}{4l^2} \right) \]  \hspace{1cm} \text{(4.5)}

\[ D_y = EI_y \text{ per unit width} = 0.13 \ E t d^2 \]  \hspace{1cm} \text{(4.6)}

\[ D_{xy} = GI_{xy} \text{ per unit width} = \frac{Gt^3}{12} \left( 1 + \frac{\pi^2 d^2}{4l^2} \right) \]  \hspace{1cm} \text{(4.7)}

where:
- \( I_x \) is the equivalent second moment of area per unit width for smeared bending normal to the corrugations;
- \( I_y \) is the equivalent second moment of area per unit width for smeared bending parallel to the corrugations;
- \( I_{xy} \) is the equivalent second moment of area per unit width for twisting.

\[ \text{NOTE: The convention for bending moments in plates relates to the direction in which the plate becomes curved, so is contrary to the convention used for beams. Bending parallel to the corrugation.} \]
(7) In circular silos, where the corrugations run circumferentially, the directions \( x \) and \( y \) in the above expressions should be taken as the meridional \( \phi \) and circumferential \( \theta \) directions respectively, see figure 1.2 (a). When the corrugations run meridionally, the directions \( x \) and \( y \) in the above expressions should be taken as the circumferential \( \theta \) and meridional \( \phi \) directions respectively.

(8) The shearing properties should be taken as independent of the corrugation orientation. The value of \( G \) may be taken as \( E / (2(1+v)) \) = 80 800 MPa.

(9) In rectangular silos, where the corrugations run horizontally, the directions \( x \) and \( y \) in the above expressions should be taken as the local axial \( x \) and horizontal \( y \) directions respectively, see figure 1.3 (a). When the corrugations run vertically or meridionally, the directions \( x \) and \( y \) in the above expressions should be interchanged on the real structure and taken as the horizontal \( y \) and axial \( x \) directions respectively.
5 Design of cylindrical walls

5.1 Basis

5.1.1 General

(1) Cylindrical steel silo walls should be so proportioned that the basic design requirements for the ultimate limit states given in section 2 are satisfied.

(2) The safety assessment of the cylindrical shell should be conducted using the provisions of EN 1993-1-6.

5.1.2 Silo wall design

(1) The cylindrical wall of the silo should be checked for the following phenomena under the limit states defined in EN 1993-1-6:
   - global stability and static equilibrium.

LS1: plastic limit state
   - resistance to bursting or rupture or plastic mechanism collapse (excessive yielding) under internal pressures or other actions;
   - resistance of joints (connections).

LS2: cyclic plastification
   - resistance to local yielding in bending;
   - local effects.

LS3: buckling
   - resistance to buckling under axial compression;
   - resistance to buckling under external pressure (wind or vacuum);
   - resistance to buckling under shear from unsymmetrical actions;
   - resistance to buckling under shear near engaged columns;
   - resistance to local failure above supports;
   - resistance to local crippling near openings;
   - resistance to local buckling under unsymmetrical actions;

LS4: fatigue
   - resistance to fatigue failure.

(2) The shell wall should satisfy the provisions of EN 1993-1-6, except where 5.3 to 5.6 provide conditions that are deemed to satisfy the provisions of that standard.

(3) For silos in Consequence Class 1, the cyclic plasticity and fatigue limit states may be ignored.

5.2 Distinctions between cylindrical shell forms

(1) For a shell wall constructed from flat rolled steel sheet, termed 'isotropic' (see figure 5.1), the resistances should be determined as defined in 5.3.2.

(2) For a shell wall constructed from corrugated steel sheets where the troughs run around the silo circumference, termed 'horizontally corrugated' (see figure 5.1), the resistances should be determined as defined in 5.3.4. For a shell wall with the troughs running up the meridian, termed 'vertically corrugated', the resistances should be determined as defined in 5.3.5.
(3) For a shell wall with stiffeners attached to the outside, termed 'externally stiffened' irrespective of the spacing of the stiffeners, the resistances should be determined as defined in 5.3.3.

(4) For a shell wall with lap joints formed by connecting adjacent plates with overlapping sections, termed 'lap-jointed' (see figure 5.1), the resistances should be determined as defined in 5.3.2.

\[ 
\begin{array}{c}
\text{Elevation} \\
\text{Plan}
\end{array} 
\]

Isotropic, externally stiffened, lap-jointed and horizontally corrugated walls

Figure 5.1: Illustrations of cylindrical shell forms

5.3 Resistance of silo cylindrical walls

5.3.1 General

(i) The cylindrical shell should satisfy the provisions of EN 1993-1-6. These may be met using the following assessments of the design resistance.

5.3.2 Isotropic welded or bolted walls

5.3.2.1 General

(1) The shell wall cross-section should be proportioned to resist failure by rupture or plastic collapse.

(2) The joints should be proportioned to resist rupture on the net section using the ultimate tensile strength.

(3) The eccentricity of lap joints should be included in the strength assessment for rupture, when relevant.

(4) The shell wall should be proportioned to resist stability failure.

5.3.2.2 Evaluation of design stress resultants

(1) Under internal pressure, frictional traction and all relevant design loads, the design stress resultants should be determined at every point in the shell using the variation in internal pressure and wall frictional traction, as appropriate.
NOTE 1: Each set of design stress resultants for stored solid loading of a silo should be based on a single set of stored solid properties.

NOTE 2: Where the design stress resultants are being evaluated to verify adequate resistance to the plastic limit state, in general the stored solid properties should be chosen to maximise the internal pressure and the condition of discharge with patch loads in EN 1991-4 should be chosen.

NOTE 3: Where the design stress resultants are being evaluated to verify adequate resistance to the buckling limit state under stored solid loads, in general the stored material properties should be chosen to maximise the axial compression and the condition of discharge with patch loads in EN 1991-4 should be chosen. However, where the internal pressure is beneficial in increasing the buckling resistance, only the filling pressures (for a consistent set of material properties) should be adopted in conjunction with the discharge axial forces, since the beneficial pressures may fall to the filling values locally even though the axial compression derives from the discharge condition.

(2) Where membrane theory is used to evaluate design stresses in the shell wall, the resistance of the shell should be adequate to withstand the highest pressure at every point.

(3) Because highly localised pressures are found to induce smaller design membrane stress resultants than would be found using membrane theory, the provisions of EN 1993-1-6 for stress design, direct design or computer design may be used to achieve a more economical design solution.

(4) Where a membrane theory analysis is used, the resulting two dimensional stress field of stress resultants \( n_{x,Ed} \), \( n_{\theta,Ed} \) and \( n_{x\theta,Ed} \) may be evaluated using the equivalent design stress:

\[
\sigma_{e,Ed} = \frac{1}{t} \sqrt{n_{x,Ed}^2 + n_{\theta,Ed}^2 - n_{x,Ed} n_{\theta,Ed} + 3n_{x\theta,Ed}^2} \quad \ldots (5.1)
\]

(5) Where an elastic bending theory analysis (LA) is used, the resulting two dimensional stress field of primary stress resultants \( n_{x,Ed} \), \( n_{\theta,Ed} \), \( n_{x\theta,Ed} \), \( m_{x,Ed} \), \( m_{\theta,Ed} \), \( m_{x\theta,Ed} \) may be transformed into the fictitious stress components:

\[
\sigma_{x,Ed} = \frac{n_{x,Ed}}{t} + \frac{m_{x,Ed}}{t^2 / 4} \quad , \quad \sigma_{\theta,Ed} = \frac{n_{\theta,Ed}}{t} + \frac{m_{\theta,Ed}}{t^2 / 4} \quad , \quad \ldots (5.2)
\]

\[
\tau_{x\theta,Ed} = \frac{n_{x\theta,Ed}}{t} + \frac{m_{x\theta,Ed}}{t^2 / 4} \quad , \quad \ldots (5.3)
\]

and the von Mises equivalent design stress:

\[
\sigma_{e,Ed} = \sqrt{\sigma_{x,Ed}^2 + \sigma_{\theta,Ed}^2 - \sigma_{x,Ed}\sigma_{\theta,Ed} + 3\tau_{x\theta,Ed}^2} \quad \ldots (5.4)
\]

NOTE: The above expressions (Ilyushin yield criterion) give a simplified conservative equivalent stress for design purposes.

### 5.3.2.3 Plastic limit state

(1) The design resistance in plates in terms of membrane stress resultants should be assessed as the equivalent stress resistance for both welded and bolted construction \( f_{e,Rd} \) given by:

\[
f_{e,Rd} = f_y / \gamma_{M0} \quad \ldots (5.5)
\]
(2) The design resistance at lap joints in welded construction $f_{c,Rd}$ should be assessed by the fictitious strength criterion:

$$f_{c,Rd} = j f_y / \gamma_{M0} \quad \ldots (5.6)$$

where $j$ is the joint efficiency factor.

(3) The joint efficiency of lap joint welded details with full continuous fillet welds should be taken as $j = j_i$. The single welded lap joint should not be used if more than 20% of the value of $\sigma_{c,Ed}$ in expression 5.4 derives from bending moments.

**NOTE:** The National Annex may choose the value of $j_i$. The recommended values of $j_i$ are given in the table below for different joint configurations.

<table>
<thead>
<tr>
<th>Joint type</th>
<th>Sketch</th>
<th>Value of $j_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double welded lap</td>
<td><img src="image1" alt="Sketch" /></td>
<td>$j_i = 1.0$</td>
</tr>
<tr>
<td>Single welded lap</td>
<td><img src="image2" alt="Sketch" /></td>
<td>$j_i = 0.35$</td>
</tr>
</tbody>
</table>

(4) In bolted construction the design resistance at net section failure at the joint should be assessed in terms of membrane stress resultants as follows:

- for meridional resistance $n_{x,Rd} = f_u t / \gamma_{M2} \quad \ldots (5.7)$
- for circumferential resistance $n_{B,Rd} = f_u t / \gamma_{M2} \quad \ldots (5.8)$
- for shear resistance $n_{A0,Rd} = 0.57 f_y t / \gamma_{M0} \quad \ldots (5.9)$

(5) The design of bolted connections should be carried out in accordance with EN 1993-1-8 or EN 1993-1-3. The effect of fastener holes should be taken into account according to EN 1993-1-1 using the appropriate requirements for tension or compression or shear as appropriate.

(6) The resistance to local loads from attachments should be dealt with as detailed in 5.4.6.

(7) At every point in the structure the design stresses should satisfy the condition:

$$\sigma_{c,Ed} \leq f_{c,Rd} \quad \ldots (5.10)$$

(8) At every joint in the structure the design stress resultants should satisfy the relevant conditions amongst:

$$n_{A,E,Ed} \leq n_{A,E,Rd} \quad \ldots (5.11)$$
5.3.2.4 Buckling under axial compression

(1) Under axial compression, the design resistance against buckling should be determined at every point in the shell using the prescribed fabrication tolerance quality of construction, the intensity of the guaranteed co-existent internal pressure, \( p \) and the circumferential uniformity of the compressive stress. The design should consider every point on the shell wall. In buckling calculations, compressive membrane forces should be treated as positive to avoid the widespread use of negative numbers.

(2) The prescribed fabrication tolerance quality of construction should be assessed as set out in table 5.1.

<table>
<thead>
<tr>
<th>Fabrication tolerance quality of construction</th>
<th>Quality parameter, ( Q )</th>
<th>Reliability class restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>16</td>
<td>Compulsory when the silo is designed to Consequence Class 1 rules</td>
</tr>
<tr>
<td>High</td>
<td>25</td>
<td>Only permitted when the silo is designed to Consequence Class 3 rules</td>
</tr>
<tr>
<td>Excellent</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The tolerance requirements for the Fabrication Tolerance Consequence Quality Classes are set out in EN 1993-1-6 and EN 1990.

(3) The representative imperfection amplitude \( w_{ok} \) should be taken as:

\[
w_{ok} = \frac{t}{Q} \sqrt{\frac{r}{t}} \quad \ldots (5.14)
\]

(4) The unpressurised elastic imperfection reduction factor \( \alpha_0 \) should be found as:

\[
\alpha_0 = \frac{0.62}{1 + 1.9 \psi \left( \frac{w_{ok}}{t} \right)^{1.44}} \quad \ldots (5.15)
\]

where the stress non-uniformity parameter \( \psi \) is unity in the case of circumferentially uniform compression, but is given in paragraph (8) for non-uniform compression.

(5) Where the silo is internally pressurised, the elastic imperfection reduction factor \( \alpha \) should be taken as the smaller of the two following values: \( \alpha_{pe} \) and \( \alpha_{pp} \) determined according to the local value of internal pressure \( p \). For silos designed to Consequence Class 1 rules, the elastic imperfection factor \( \alpha \) should not be taken as greater than \( \alpha = \alpha_{c} \).
(6) The elastic pressurised imperfection reduction factor $\alpha_{pe}$ should be based on the smallest local internal pressure (a value that can be guaranteed to be present) at the location of the point being assessed, and coexistent with the axial compression:

$$\alpha_{pe} = \alpha_0 + (1 - \alpha_0) \left( \frac{\bar{p}_s}{\bar{p}_s + 0.3 \sqrt{\alpha_0}} \right)$$  \hspace{1cm} ... (5.16)

with:

$$\bar{p}_s = \frac{p_{s,r}}{t \sigma_{s,rcr}}$$  \hspace{1cm} ... (5.17)

where:

- $p_{s,r}$ is the minimum reliable design value of local internal pressure (see EN 1991-4);
- $\sigma_{s,rcr}$ is the elastic critical buckling stress (see expression 5.28).

(7) The plastic pressurised imperfection reduction factor $\alpha_{pp}$ should be based on the largest local internal pressure at the location of the point being assessed, and coexistent with the axial compression:

$$\alpha_{pp} = \left[ 1 - \left( \frac{p_g}{\lambda_i} \right)^2 \right]^{1 - \frac{1}{1 + 12 + s^2/2}} \left[ \frac{1}{s(s+1)} \left( \frac{\lambda_i^2 + 1.2 \lambda_i^3}{s(s+1)} \right) \right]$$  \hspace{1cm} ... (5.18)

with:

$$\bar{p}_g = \frac{p_g}{\sigma_{s,rcr}}, \frac{r}{t}$$  \hspace{1cm} ... (5.19)

$$s = \left( \frac{1}{400} \right) \left( \frac{r}{t} \right)$$  \hspace{1cm} ... (5.20)

$$\lambda_i^2 = \frac{f_y}{\sigma_{s,rcr}}$$  \hspace{1cm} ... (5.21)

where:

- $p_g$ is the largest design value of the local internal pressure (see EN 1991-4).

(8) Where the axial compression stress is non-uniform around the circumference, the effect should be represented by the stress non-uniformity parameter $\psi$, which should be determined from the linear elastic stress distribution of acting axial compressive stress distribution. The axial compressive membrane stress distribution around the circumference at the chosen level should be transformed as shown in figure 5.2. The design value of axial compressive membrane stress $\sigma_{s,Ed}$ at the most highly stressed point at this axial coordinate is denoted as $\sigma_{s,Ed}$. 

40
The design value of axial compressive membrane stress at a second point, at the same axial coordinate, but separated from the first point by the circumferential distance

\[ y = r \Delta \theta = 4 \sqrt{r t} \]  \hspace{1cm} (5.22)

should be taken as \( \sigma_{x1,Ed} \).

(9) Where the stress ratio

\[ s = \left( \frac{\sigma_{x1,Ed}}{\sigma_{x0,Ed}} \right) \]  \hspace{1cm} (5.23)

lies in the range \( 0.3 < s < 1.0 \), the above location for the second point is satisfactory. Where the value of \( s \) lies outside this range, an alternative value of \( r \Delta \theta \) should be chosen so that the value of \( s \) is found to be approximately \( s = 0.5 \). The following calculation should then proceed with a matched pair of values of \( s \) and \( \Delta \theta \).

![Figure 5.2: Representation of local distribution of axial membrane stress resultant around the circumference](image)

(10) The equivalent harmonic \( j \) of the stress distribution should be obtained as:

\[ j = 0.25 \sqrt{\frac{r}{t}} \arccos \left( \frac{\sigma_{x1,Ed}}{\sigma_{x0,Ed}} \right) \]  \hspace{1cm} (5.24)

and the stress non-uniformity parameter \( \psi \) should be determined as:

\[ \psi = \frac{1 - b_1 j}{1 + b_1 j} \]  \hspace{1cm} (5.25)

with:
\[
b_1 = 0.5 \sqrt{\frac{t}{r}} \quad \ldots \text{(5.26)}
\]

\[
b_2 = \frac{(1 - b_1)}{\psi_b} - 1 \quad \ldots \text{(5.27)}
\]

where \( \psi_b \) is the value of stress non-uniformity parameter under global bending conditions.

**NOTE:** The National Annex may choose the value of \( \psi_b \). The value \( \psi_b = 0.40 \) is recommended.

(11) The equivalent harmonic \( j \) at which imperfections cause no reduction below the uniform compression critical buckling resistance may be taken as \( j_o = 1/b_1 \). Where it is found that \( j > j_o \), the value of \( j \) should be taken as \( j = j_o \).

(12) Where a horizontal lap joint is used, causing eccentricity of the axial force in passing through the joint, the value of \( \alpha \) given in paragraphs (4) to (7) above should be reduced to \( \alpha_L \) if the eccentricity of the middle surface of the plates to one another exceeds \( k_1 t \) and the change in plate thickness at the joint is not more than \( k_2 t \), where \( t \) is the thickness of the thinner plate at the joint. Where the eccentricity is smaller than this value, or the change in plate thickness is greater, no reduction need be made in the value of \( \alpha \).

**NOTE 1:** The National Annex may choose the values of \( \alpha_L \), \( k_1 \) and \( k_2 \). The values \( \alpha_L = 0.7 \alpha \), \( k_1 = 0.5 \) and \( k_2 = 0.25 \) are recommended, where \( \alpha \) is given by \( \alpha_p \) or \( \alpha_pp \) as appropriate.

**NOTE 2:** The buckling strength is only reduced below the value that would otherwise apply if the lower course is not thick enough to restrain the formation of a weaker buckle when an imperfection occurs immediately above the lap joint.

(13) The critical buckling stress of the isotropic wall should be calculated as:

\[
\sigma_{x,cr} = \frac{E}{\sqrt{3(1-\nu^2)}} \cdot \frac{t}{r} = 0.605E \cdot \frac{t}{r} \quad \ldots \text{(5.28)}
\]

(14) The characteristic buckling stress should be found, using the appropriate value of \( \alpha \) from paragraphs (4), (5), (6), (7) and (8) above as:

\[
\sigma_{x,Rk} = \chi_x f_y \quad \ldots \text{(5.29)}
\]

**NOTE:** The special convention using \( \sigma_{Rk} \) and \( \sigma_{Rd} \) for characteristic and design buckling resistances follows that of prEN1993-1-6 for shell structures and differs from that detailed in EN1993-1-1.

(15) The buckling reduction factor \( \chi_x \) should be determined as a function of the relative slenderness of the shell \( \bar{\lambda}_v \) from:

\[
\chi_x = 1 \quad \text{when} \quad \bar{\lambda}_v \leq \bar{\lambda}_v' \quad \ldots \text{(5.30)}
\]
\[ \chi_x = 1 - \beta \left( \frac{\lambda_0 - \lambda_{n0}}{\lambda_p - \lambda_{n0}} \right)^\eta \quad \text{when} \quad \lambda_0 < \lambda_x < \lambda_p \quad \ldots \ (5.31) \]

\[ \chi_x = \frac{\alpha}{\lambda_x^2} \quad \text{when} \quad \lambda_p \leq \lambda_x \quad \ldots \ (5.32) \]

with:

\[ \lambda_x = \frac{f}{\sigma_{x,kr}} \quad \ldots \ (5.33) \]

\[ \lambda_0 = 0.2 \quad \ldots \ (5.34) \]

\[ \lambda_p = \frac{\alpha}{\sqrt{1 - \beta}} \quad \ldots \ (5.35) \]

where \( \alpha \) is chosen as the value of \( \alpha_p, \alpha_{pe}, \alpha_{pp} \) or \( \alpha_L \) as appropriate.

**NOTE:** The National Annex may choose the values of \( \beta \) and \( \eta \). The values \( \beta = 0.60 \) and \( \eta = 1.0 \) are recommended.

(16) The design buckling membrane stress should be determined as:

\[ \sigma_{x,rd} = \sigma_{x,kr} / \chi_{M1} \quad \ldots \ (5.36) \]

where \( \chi_{M1} \) is given in 2.9.2.

(17) At every point in the structure the design stress resultants should satisfy the condition:

\[ n_{x,Ed} \leq \tau \sigma_{x,kg} \quad \ldots \ (5.37) \]

(18) Where the wall contains a lap joint satisfying the conditions defined in (12), the measurement of the maximum permissible measurable imperfection need not be taken across the lap joint itself.

(19) The design of the shell against buckling under axial compression above a local support, near a bracket (e.g. to support a conveyor gantry), and near an opening should be undertaken as stipulated in 5.6.

### 5.3.2.5 Buckling under external pressure, internal partial vacuum and wind

(1) The buckling assessment should be carried out using EN 1993-1-6, but these may be met using the following assessments of the design resistance.

(2) The lower edge of the cylindrical shell should be effectively anchored to resist vertical displacements, see 5.4.7.

(3) Under wind or partial vacuum, the silo wall should be broken into segments lying between stiffening rings or changes of plate thickness or boundary conditions.
(4) A buckling assessment should be carried out on each segment or potential group of segments where a buckle could form, including the thinnest segment and adding others progressively. The lowest design buckling pressure should be found from these alternative assessments.

(5) The critical buckling external pressure for an isotropic wall should be found as:

\[ p_{n,Rcu} = 0.92 \, C_w \, C_b \, E \, \left( \frac{r}{t} \right) \left( \frac{t}{\ell} \right)^{2.5} \]  \hspace{1cm} (5.38)

where:
- \( t \) is the thickness of the thinnest part of the wall;
- \( \ell \) is the height between stiffening rings or boundaries;
- \( C_b \) is the external pressure buckling coefficient;
- \( C_w \) is the wind pressure distribution coefficient.

(6) The parameter \( C_b \) should be evaluated based on the condition at the upper edge according to table 5.2.

<table>
<thead>
<tr>
<th>Upper edge condition</th>
<th>Roof integrally structural connected to wall (continuous)</th>
<th>Upper edge ring satisfying 5.3.2.5 (12)-(14)</th>
<th>Upper edge not satisfying 5.3.2.5 (12)-(14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_b )</td>
<td>1.0</td>
<td>1.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

(7) Where the silo is in a close-spaced silo group, the wind pressure distribution coefficient (relating to the pressure at the windward generator of the silo) should be taken as \( C_w = 1.0 \).

(8) Where the silo is isolated and subject only to wind loading, the wind pressure distribution coefficient (relating to the pressure at the windward generator of the silo) should be taken as the greater of:

\[ C_w = \frac{2.2}{1 + 0.1 \sqrt{C_b \frac{r}{\ell} \sqrt{\frac{r}{t}}}} \]  \hspace{1cm} (5.39)

\[ C_w = 1.0 \]  \hspace{1cm} (5.40)

(9) Where the silo is isolated and a combination of wind loading and internal vacuum exist, the value of \( C_w \) should be determined as a linear combination of 1.0 and the calculated value given in (8), according to the proportions of the external pressure that arise from each source.

(10) The design maximum external pressure (windward generator) under wind and/or partial vacuum should be assessed as:

\[ p_{n,\text{Rd}} = \alpha_n \, p_{n,Rcu} / \gamma_{M1} \]  \hspace{1cm} (5.41)

where \( \alpha_n \) is the elastic buckling imperfection reduction factor and \( \gamma_{M1} \) is given in 2.9.2.
NOTE: The National Annex may choose the value of $\alpha_n$. The value $\alpha_n = 0.5$ is recommended.

(11) The resistance check should satisfy the condition:

$$P_{n,Ed} \leq P_{n,Rd}$$  \hspace{1cm} \text{(5.42)}

where:

- $P_{n,Ed}$ is the design value of the maximum external pressure under wind and/or partial vacuum.

(12) For the upper edge of a cylinder to be treated as effectively restrained by a ring, the ring should satisfy both a strength and a stiffness requirement. Unless a more thorough assessment is made using numerical analysis, the design value of the circumferential (hoop) force and circumferential bending moment about a vertical axis in the ring should be taken as:

$$N_{\theta,Ed} = 0.5 r L P_{n,Ed}$$  \hspace{1cm} \text{(5.43)}

$$M_{\theta,Ed} = M_{\theta,Edo} + M_{\theta,Edw}$$  \hspace{1cm} \text{(5.44)}

with:

$$M_{\theta,Edo} = 0.0033 P_{nS1} r^2 L \left( \frac{P_{nS1}}{P_{nS1} - P_{n,Edo}} \right)$$  \hspace{1cm} \text{(5.45)}

$$M_{\theta,Edw} = 0.17 P_{n,Edw} r^2 L \left( \frac{P_{n,Edw}}{P_{nS1} - P_{n,Edo}} \right)$$  \hspace{1cm} \text{(5.46)}

$$P_{nS1} = \frac{6EI_z}{r^3L}$$  \hspace{1cm} \text{(5.47)}

where:

- $P_{n,Edo}$ is the design value of the uniform component of the external pressure under wind and/or partial vacuum;
- $P_{n,Edw}$ is the design value of the stagnation point pressure under wind;
- $P_{nS1}$ is the reference pressure for ring bending moment evaluations;
- $M_{\theta,Edo}$ is the design value of the bending moment associated with out-of-roundness;
- $M_{\theta,Ed}$ is the design value of the bending moment due to wind;
- $I_z$ is the second moment of area of the ring for circumferential bending;
- $L$ is the total height of the shell wall;
- $t$ is the thickness of the thinnest strake.

(13) Where the ring at the upper edge of a cylinder is made as a cold formed construction, the value of $M_{\theta,Edo}$ should be increased by 15% above that given by expression 5.45.
(14) The flexural rigidity $EI_y$ of a ring at the upper of the cylinder about its vertical axis (circumferential bending) should exceed the larger of:

$$EI_{y,\text{min}} = k_1 E L r^3$$

and

$$EI_{y,\text{min}} = 0.08 C_w E r r^3 \sqrt{(r/l)}$$

where $C_w$ is the wind pressure distribution coefficient given in (7) or (8).

**NOTE:** The National Annex may choose the value of $k_1$. The value $k_1 = 0.1$ is recommended.

### 5.3.2.6 Membrane shear

(1) Where a major part of the silo wall is subjected to shear loading (as with eccentric filling, earthquake loading etc.), the membrane shear buckling resistance should be taken as that for a shell in torsion at each horizontal level. The axial variation in shear may be taken into account in design.

(2) The critical shear buckling stress of the isotropic wall should be calculated as:

$$\tau_{\theta, R\text{cr}} = 0.75 E \left( \frac{r}{l} \right)^{0.5} \left( \frac{r}{l} \right)^{1.25}$$

where:

- $t$ is the thickness of the thinnest part of the wall;
- $\ell$ is the height between stiffening rings or boundaries.

(3) A stiffening ring which is required as the boundary for a shear buckling zone should have a flexural rigidity $EI_y$ about the axis for bending around the circumference not less than:

$$EI_{y,\text{min}} = k_s E r^3 \sqrt{r/l}$$

where the values of $l$ and $t$ are taken as the same as those used in the most critical buckling mode in paragraph (2).

**NOTE:** The National Annex may choose the value of $k_s$. The value $k_s = 0.10$ is recommended.

(4) Where the shear $\tau$ varies linearly with height in the structure, the critical shear buckling resistance at the point of highest shear may be increased to:

$$\tau_{\theta, R\text{cr}} = 1.4 E \left( \frac{r}{l_0} \right)^{0.5} \left( \frac{r}{l} \right)^{1.25}$$

with $l_0$ determined from:

$$l_0 = \frac{\tau_{\theta, R\text{cr}, \text{max}}}{\left( \frac{d\tau_{\theta, R\text{cr}}}{dx} \right)}$$
where \( \left( \frac{d \tau_{\theta,\text{Ed}}}{dx} \right) \) is the axial rate of change of shear with height averaged over the zone and \( \tau_{\theta,\text{Ed},\text{max}} \) is the peak value of shear stress. Where the length \( l_0 \) exceeds the height of the structure, this rule should not be used, but the shell should be treated as subject to uniform membrane shear set out in (2).

(5) Where local shear stresses are induced by local supports and load-bearing axial stiffeners, the critical shear buckling resistance, assessed in terms of the local value of the shear transfer between the axial stiffener and the shell may be evaluated at the point of highest shear as:

\[
\tau_{\theta,\text{Rcr}} = 1.4 E \left( \frac{r}{l_0} \right)^{0.5} \left( \frac{t}{r} \right)^{1.25}
\]  

... (5.54)

in which \( l_0 \) is found as:

\[
E_0 = \frac{\tau_{\theta,\text{Ed,max}}}{d \tau_{\theta,\text{Ed}}/dy} \tag{5.55}
\]

(5.55) OE

where \( \left( \frac{d \tau_{\theta,\text{Ed}}}{dy} \right) \) is the circumferential rate of change of shear with distance from the stiffener averaged over the zone, and \( \tau_{\theta,\text{Ed},\text{max}} \) is the peak value of shear stress.

(6) The design buckling stress should be determined as the lesser of:

\[
\tau_{\theta,\text{Rd}} = \alpha \tau_{\theta,\text{Rcr}} / \gamma_{\text{M1}}
\]  

... (5.56)

and

\[
\tau_{\theta,\text{Rd}} = 0.57 f_\tau / \gamma_{\text{M1}}
\]  

... (5.57)

where:

- \( \alpha \) is the elastic buckling imperfection reduction factor;
- \( \gamma_{\text{M1}} \) is the partial factor given in 2.9.2.

NOTE: The National Annex may choose the value of \( \alpha \). The value \( \alpha = 0.80 \) is recommended.

(7) At every point in the structure the design stress resultants should satisfy the condition:

\[
n_{\theta,\text{Ed}} \leq t \tau_{\theta,\text{Rd}}
\]  

... (5.58)

### 5.3.2.7 Interactions between meridional compression, circumferential compression and membrane shear

(1) Where the stress state in the silo wall contains significant components of more than one compressive membrane stress or shear stress, the provisions of EN 1993-1-6 should be followed.

(2) The requirements of this interaction may be ignored if all but one of the design stress components are less than 20% of the corresponding buckling design resistance.
5.3.2.8 Fatigue, LS4

(1) For silos in Consequence Class 3, the provisions of EN 1993-1-6 should be followed.

(2) For silos in Consequence Class 2, a fatigue check should be carried out if the design life of the structure involves more than \( N_r \) cycles of filling and discharge.

**NOTE:** The National Annex may choose the value of \( N_r \). The value \( N_r = 10000 \) is recommended.

5.3.2.9 Cyclic plasticity, LS2

(1) For silos in Consequence Class 3, the provisions of EN 1993-1-6 should be followed. A check for failure under cyclic plasticity should be made at discontinuities, near local ring stiffeners and near attachments.

(2) Silos in other Consequence Classes, this check may be omitted.

5.3.3 Isotropic walls with vertical stiffeners

5.3.3.1 General

(1) Where an isotropic wall is stiffened by vertical (stringer) stiffeners, the effect of compatibility of the shortening of the wall due to internal pressure should be taken into account in assessing the vertical compressive stress in both the wall and the stiffeners.

(2) The design stress resultants, resistances and checks should be carried out as in 5.3.2, but including the additional provisions set out here.

5.3.3.2 Plastic limit state

(1) The resistance against rupture on a vertical seam should be determined as for an isotropic shell (5.3.2).

(2) Where a structural connection detail includes the stiffener as part of the means of transmitting circumferential tensions, the effect of this tension on the stiffener should be taken into account in evaluating the force in the stiffener and its susceptibility to rupture under circumferential tension.

5.3.3.3 Buckling under axial compression

(1) The wall should be designed for the same axial compression buckling criteria as the unstiffened wall unless the stiffeners are at closer spacings than \( 2\sqrt{t} \), where \( t \) is the local thickness of the wall.

(2) Where vertical stiffeners are placed at closer spacings than \( 2\sqrt{t} \), the buckling resistance of the complete wall should be assessed either by assuming that paragraph (1) above applies, or by using the global analysis procedures of EN 1993-1-6.

(3) The axial compression buckling strength of the stiffeners themselves should be evaluated using the provisions of EN 1993-1-1 or EN 1993-1-3 (cold formed steel members) or EN 1993-1-5 as appropriate.

(4) The eccentricity of the stiffener to the shell wall should be taken into account.

5.3.3.4 Buckling under external pressure, partial vacuum or wind

(1) The wall should be designed for the same external pressure buckling criteria as the unstiffened wall unless a more rigorous calculation is necessary.
Where a more rigorous calculation is needed, the vertical stiffeners may be smeared to give an orthotropic wall, and the buckling stress assessment carried out using the provisions of 5.3.4.5, with \( C_\theta = C_\theta = Et \) and \( C_{\theta h} = 0.38 Et \).

### 5.3.3.5 Membrane shear

1. Where a major part of the silo wall is subjected to shear loading (as with eccentric filling, earthquake loading etc.), the membrane shear buckling resistance should be found as for an isotropic unstiffened wall (see 5.3.2.6), but the calculated resistance may be increased if account is taken of the effect of the stiffeners. The equivalent length \( l' \) of shell in shear may be taken as the lesser of the height between stiffening rings or boundaries and twice the horizontal separation of the vertical stiffeners, provided that each stiffener has a flexural rigidity \( EI_y \) for bending in the vertical direction (about a circumferential axis) greater than:

\[
EI_{y,\text{min}} = k_s E r^3 \sqrt{r\ell}
\]

where the values of \( \ell \) and \( t \) are taken as the same as those used in the most critical buckling mode.

**NOTE:** The National Annex may choose the value of \( k_s \). The value \( k_s = 0.10 \) is recommended.

2. Where a discrete stiffener is abruptly terminated part way up the shell, the force in the stiffener should be taken to be uniformly redistributed into the shell over a length not exceeding \( k_i \sqrt{rt} \).

**NOTE:** The National Annex may choose the value of \( k_i \). The value \( k_i = 4.0 \) is recommended.

3. Where the stiffeners are terminated as in (2), or used to introduce local forces into the shell, the assessed resistance for shear transmission between the stiffener and the shell should not exceed the value given in 5.3.2.6 for linearly varying shear.

### 5.3.4 Horizontally corrugated walls

#### 5.3.4.1 General

1. All calculations should be carried out with thicknesses exclusive of coatings and tolerances.

2. The minimum steel core thickness for the corrugated sheeting of the wall should meet the requirements of EN 1993-1-3. In bolted construction, the bolt size should not be less than M8.

3. Where the cylindrical wall is fabricated from corrugated sheeting with the corrugations running horizontally and vertical stiffeners are attached to the wall, the corrugated wall should be assumed to carry no vertical forces unless the wall is treated as an orthotropic shell, see 5.3.4.3.3.

4. Particular attention should be paid to ensure that the stiffeners are flexurally continuous with respect to bending in the meridional plane normal to the wall, because the flexural continuity of the stiffener is essential in developing resistance to buckling under wind or external pressure as well as when the stored solids flow.

5. Where the wall is stiffened with vertical stiffeners, the fasteners between the sheeting and stiffeners should be proportioned to ensure that the distributed shear loading from stored solids (frictional traction) on each part of the wall sheeting is transferred into the stiffeners. The sheeting thickness should be chosen to ensure that local rupture at these fasteners is prevented, taking proper account of the reduced bearing strength of fasteners in corrugated sheeting.
(6) The design stress resultants, resistances and checks should be carried out as in 5.3.2, but including the additional provisions set out in (1) to (5) above.

\[\text{NOTE: More detailed information on the design of corrugated silos is available in the references given in Annex D.}\]

\[\text{Note deleted}\]

5.3.4.2 Plastic limit state

(1) Bolts for fastenings between panels should satisfy the requirements of EN 1993-1-8.

(2) The joint detail between panels should comply with the provisions of EN 1993-1-3 for connections in tension or compression.

(3) The spacing between fasteners around the circumference should not exceed 3° of the circumference.

\[\text{NOTE: A typical bolt arrangement detail for a panel is shown in figure 5.4.}\]
Where penetrations are made in the wall for hatches, doors, augers or other items, a thicker corrugated sheet should be used locally to ensure that the local stress raisers associated with mismatches of stiffness do not lead to local rupture.

5.3.4.3 Buckling under axial compression

5.3.4.3.1 General

Under axial compression, the design resistance should be determined at every point in the shell using the prescribed fabrication tolerance quality of construction, the intensity of the guaranteed co-existant internal pressure $p$ and the circumferential uniformity of the compressive stress. The design should consider every point on the shell wall.

If the horizontally corrugated wall is stiffened with vertical stiffeners, the buckling design of the wall should be carried out using one of two alternative methods:

a) buckling of the equivalent orthotropic shell (following 5.3.4.3.3) if the horizontal distance between stiffeners satisfies 5.3.4.3.3 (2);

b) buckling of the individual stiffeners (corrugated wall assumed to carry no axial force, but providing restraint to the stiffeners) and following 5.3.4.3.4 if the horizontal distance between stiffeners does not satisfy 5.3.4.3.3 (2).

5.3.4.3.2 Unstiffened wall

If the corrugated shell has no vertical stiffeners, the characteristic value of local plastic buckling resistance should be determined as the greater of:

$$n_{s,\text{pl}} = \frac{t^2 f_y}{2d}$$

... (5.60)

and

$$n_{s,\text{pl}} = R_\theta \frac{t}{r^*} f_y$$

... (5.61)

where:
The local plastic buckling resistance $n_{x,Rk}$ should be taken as independent of the value of internal pressure $p_n$.

**NOTE:** The local plastic buckling resistance is the resistance to corrugation collapse or “roll-down”.

(2) The design value of the local plastic buckling resistance should be determined as:

$$n_{x,Rd} = \alpha_x \frac{n_{x,Rk}}{\gamma_{M0}}$$

where:

- $\alpha_x$ is the elastic buckling imperfection reduction factor;
- $\gamma_{M0}$ is the partial factor given in 2.9.2.

**NOTE:** The National Annex may choose the value of $\alpha_x$. The value $\alpha_x = 0.80$ is recommended.

(3) At every point in the structure the design stress resultants should satisfy the condition:

$$n_{x,Ed} \leq n_{x,Rd}$$

**5.3.4.3.3 Stiffened wall treated as an orthotropic shell**

(1) If the wall is treated as an orthotropic shell (method (a) in 5.3.4.3.1), the stiffnesses of the sheeting in different directions should be taken from 4.4. The resulting smeared stiffnesses should be taken to be uniformly distributed. The equivalent shell middle surface should be taken as the central axis from which the amplitude is measured (see Fig. 4.2).

(2) The horizontal distance between stiffeners $d_s$ should not be more than $d_{s,max}$ given by:

$$d_{s,max} = k_{dx} \left( \frac{r^2 D_y}{C_y} \right)^{0.25}$$

where:

- $D_y$ is the flexural rigidity per unit width of the thinnest sheeting parallel to the corrugations;
- $C_y$ is the stretching stiffness per unit width of the thinnest sheeting parallel to the corrugations;
- $r$ is the cylinder radius.

**NOTE:** The National Annex may choose the value of $k_{dx}$. The value $k_{dx} = 7.4$ is recommended.
(3) The critical buckling stress resultant $n_{x,Rcr}$ per unit circumference of the orthotropic shell (method (a) in 5.3.4.3.1) should be evaluated at each appropriate level in the silo by minimising the following expression with respect to the critical circumferential wave number $j$ and the buckling height $\ell_i$:

$$n_{x,Rcr} = \frac{1}{j^2 \omega^2} \left( A_1 + \frac{A_2}{A_3} \right)$$ ... (5.65)

with:

$$A_1 = j^4 \left[ \omega^4 C_{44} + 2 \omega^2 (C_{45} + C_{66}) + C_{55} \right] + C_{22} + 2 j^2 C_{25}$$ ... (5.66)

$$A_2 = 2 \omega^2 (C_{12} + C_{33}) (C_{22} + j^2 \omega^2 C_{14})$$

$$- (\omega^2 C_{11} + C_{33}) (C_{22} + j^2 \omega^2 C_{14})^2 - \omega^2 (C_{22} + \omega^2 C_{33}) (C_{12} + j^2 \omega^2 C_{14})^2$$ ... (5.67)

$$A_3 = (\omega^2 C_{11} + C_{33}) (C_{22} + C_{25} + \omega^2 C_{33}) - \omega^2 (C_{12} + C_{33})^2$$ ... (5.68)

with:

$$C_{11} = C_\theta + E A_s / d_s, \quad C_{22} = C_\theta + E A_r / d_r$$

$$C_{12} = \sqrt{C_\theta C_\phi}, \quad C_{33} = C_\phi$$

$$C_{14} = e_s E A_s / (r d_s), \quad C_{25} = e_r E A_r / (r d_r)$$

$$C_{44} = [D_\theta + E I_s / d_s + E A_s e_s^2 / d_s] / r^2, \quad C_{55} = [D_\theta + E I_s / d_s + E A_s e_s^2 / d_s] / r^2$$

$$C_{45} = \sqrt{D_\theta D_\phi} / r^2, \quad C_{66} = [D_\phi + 0.5 (G I_s / d_s + G I_r / d_r)] / r^2$$

$$\omega = \frac{\pi r}{j \ell_i}$$

where:

- $\ell_i$ is the half wavelength of the potential buckle in the vertical direction;
- $A_s$ is the cross-sectional area of a stringer stiffener;
- $I_s$ is the second moment of area of a stringer stiffener about the circumferential axis (vertical bending);
- $d_s$ is the separation between stringer stiffeners;
- $I_{ts}$ is the uniform torsion constant of a stringer stiffener;
- $e_s$ is the outward eccentricity from the shell middle surface of a stringer stiffener;
- $A_r$ is the cross-sectional area of a ring stiffener;
- $I_r$ is the second moment of area of a ring stiffener about the vertical axis (circumferential bending);
- $d_r$ is the separation between ring stiffeners;
- $I_{tr}$ is the uniform torsion constant of a ring stiffener;
is the outward eccentricity from the shell middle surface of a ring stiffener;

$C_\theta$ is the sheeting stretching stiffness in the circumferential direction (see 4.4 (5) and (7));

$D_\theta$ is the sheeting flexural rigidity in the circumferential direction (see 4.4 (6) and (7));

$D_{\theta\theta}$ is the sheeting flexural rigidity in the axial direction (see 4.4 (5) and (7));

$r$ is the radius of the silo.

NOTE 1: The above properties for the stiffeners ($A, I, I_e$ etc.) relate to the stiffener section alone: no allowance can be made for an “effective” section including parts of the shell wall.

NOTE 2: The lower boundary of the buckle can be taken at the point at which either the sheeting thickness changes or the stiffener cross-section changes; the buckling resistance at each such change needs to be checked independently.

(5) The design buckling resistance $n_{x,Rd}$ for the orthotropic shell (method (a) in 5.3.4.3.1) should be determined as the lesser of:

$$n_{x,Rd} = \alpha_x n_{x,Rcr} \gamma_{M1}$$

and

$$n_{x,Rd} = A_{eff} f_y / (d_s \gamma_{M0})$$

where:

$d_s$ is the distance between the stringer stiffeners;

$A_{eff}$ is the effective cross-sectional area of the stringer stiffener;

$\alpha_x$ is the elastic buckling imperfection reduction factor;

$\gamma_{M1}$ is the partial factor given in 2.9.2.

NOTE: The National Annex may choose the value of $\alpha_x$. The value $\alpha_x = 0.80$ is recommended.

(6) At every point in the structure the design stress resultants should satisfy the condition:

$$n_{x,Ed} \leq n_{x,Rd}$$

5.3.4.3.4 Stiffened wall treated as carrying axial compression only in the stiffeners

(1) If the corrugated sheeting is assumed to carry no axial force (method (b) in 5.3.4.3.1), the sheeting may be assumed to restrain all buckling displacements of the stiffener in the plane of the wall, and the resistance to buckling should be calculated using one of the two following alternative methods:

a) ignoring the supporting action of the sheeting in resisting buckling displacements normal to the wall;

b) allowing for the stiffness of the sheeting in resisting buckling displacements normal to the wall.
(2) Using method (a) in (1), the resistance of an individual stiffener may be taken as the resistance to concentric compression on the stiffener. The design buckling resistance $N_{b,Rd}$ should be obtained from:

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$$  \hspace{1cm} \text{(5.72)}

where:

- $A_{eff}$ is the effective cross-sectional area of the stiffener.

The reduction factor $\chi$ should be obtained from EN 1993-1-1 for flexural buckling normal to the wall (about the circumferential axis) using buckling curve $c$ irrespective of the section adopted (imperfection factor $\alpha = 0.49$). The effective length of column used in determining the reduction factor $\chi$ should be taken as the distance between adjacent ring stiffeners.

(3) If the elastic restraint provided by the wall against buckling of the stiffener is taken into account, both of the following conditions should be met:

a) The section of wall deemed to provide restraint should be the length of wall as far as the adjacent stiffeners (see figure 5.5), with simply supported conditions at the two ends.

b) No account should be taken of the possible stiffness of the stored bulk solid.

(4) Unless more precise calculations are made, the critical buckling resistance $N_{b,Rc}$ should be calculated assuming uniform compression on the cross-section at any level, as the lesser of the two expressions:

$$N_{b,Rc} = 2\sqrt{EJ/K}$$  \hspace{1cm} \text{(5.73)}

$$N_{b,Rc} = \frac{A_{eff} f_y}{\gamma_{M1}}$$  \hspace{1cm} \text{(5.74)}

where:

- $EJ$ is the flexural rigidity of the stiffener for bending out of the plane of the wall (Nmm²);
- $K$ is the flexural stiffness of the sheeting (N/mm per mm of wall height) spanning between vertical stiffeners, as indicated in figure 5.5;
- $A_{eff}$ is the effective cross-sectional area of the stiffener.

(5) The flexural stiffness of the wall plate $K$ should be determined assuming that the sheeting spans between adjacent vertical stiffeners on either side with simply supported boundary conditions, see figure 5.5. The value of $K$ may be estimated as:

$$K = k_s \frac{D_s}{d_s^3}$$  \hspace{1cm} \text{(5.75)}

where:

- $D_s$ is the flexural rigidity of the sheeting for circumferential bending;
- $d_s$ is the separation of the vertical stiffeners.
If the corrugation is an arc-and-tangent or sinusoidal profile, the value of $D_y$ may be taken from 4.4 (6). If other corrugation sections are adopted, the flexural rigidity for circumferential bending should be determined from first principles.

**NOTE:** The National Annex may choose the value of $k_d$. The value $k_d = 6$ is recommended.

(6) At every point in the stiffener, the design forces should satisfy the condition:

$$N_{b,Ed} \leq N_{b,Rd} \quad \ldots (5.76)$$

![Figure 5.5: Evaluation of restraint stiffness against stiffener column buckling](image)

**5.3.4.4 Local, distortional and flexural torsional failure of stiffeners**

(1) The resistance of the stiffeners to local, distortional and flexural torsional buckling should be determined using EN 1993-1-3 (cold formed construction).

**5.3.4.5 Buckling under external pressure, partial vacuum or wind**

(1) The equivalent membrane and flexural properties of the sheeting should be found using 4.4.

(2) The bending and stretching properties of the ring and stringer stiffeners, and the outward eccentricity of the centroid of each from the middle surface of the shell wall should be determined, together with the separation between the stiffeners $d_s$.

(3) The horizontal distance between stiffeners $d_s$ should not be more than $d_{s,\text{max}}$ given by:

$$d_{s,\text{max}} = k_d \left( \frac{r^2 D_y}{C_y} \right)^{0.25} \quad \ldots (5.77)$$

where:

$D_y$ is the flexural rigidity per unit width of the thinnest sheeting parallel to the corrugations;

$C_y$ is the stretching stiffness per unit width of the thinnest sheeting parallel to the corrugations;

$r$ is the cylinder radius.

**NOTE:** The National Annex may choose the value of $k_d$. The value $k_d = 7.4$ is recommended.

(4) The critical buckling stress for uniform external pressure $P_{n,Rcu}$ should be evaluated by minimising the following expression with respect to the critical circumferential wave number, $j$:

$$P_{n,Rcu} = \frac{1}{r j^2} \left( A_1 + \frac{A_2}{A_3} \right) \quad \ldots (5.78)$$
(5) Where the stiffeners or sheeting change with height up the wall, several potential buckling lengths \( \xi_i \) should be examined to determine which is the most critical, assuming always that the upper end of a buckle is at the top of the zone of thinnest sheeting.

**NOTE:** If a zone of thicker sheeting is used above the zone that includes the thinnest sheeting, the upper end of the potential buckle could occur either at the top of the thinnest zone, or at the top of the wall.

(6) Unless more precise calculations are made, the thickness assumed in the above calculation should be taken as the thickness of the thinnest sheeting throughout.

(7) Where the silo has no roof and is potentially subject to wind buckling, the above calculated pressure should be reduced by the factor 0.6.

(8) The design buckling stress for the wall should be determined using the procedure given in 5.3.2.5, with \( C_b = C_w = 1.0 \) and taking \( \alpha_n = 0.5 \), but adopting the critical buckling pressure \( P_{n,Rcu} \) from (4) above.

### 5.3.4.6 Membrane shear

(1) The buckling resistance of the shell under membrane shear should be determined using the provisions of EN 1993-1-6.
5.3.5 Vertically corrugated walls with ring stiffeners

5.3.5.1 General

(1) If the cylindrical wall is fabricated using corrugated sheeting with the corrugations running vertically, both of the following conditions should be met:
   a) The corrugated wall should be assumed to carry no horizontal forces.
   b) The corrugated sheeting should be assumed to span between attached rings, using the centre to centre separation between rings, and adopting the assumption of sheeting continuity.

(2) The joints between sheeting sections should be designed to ensure that assumed flexural continuity is achieved.

(3) The evaluation of the axial compression force in the wall arising from wall frictional tractions from the bulk solid should take account of the full circumference of the silo, allowing for the profile shape of the corrugation.

(4) If the corrugated sheeting extends to a base boundary condition, the local flexure of the sheeting near the boundary should be considered, assuming a radially restrained boundary.

(5) The design stress resultants, resistances and checks should be carried out as in 5.3.2, but including the additional provisions set out in 5.3.5.2 to 5.3.5.5.

5.3.5.2 Plastic limit state

(1) In checking the plastic limit state, the corrugated wall should be assumed to carry no circumferential forces.

(2) The spacing of ring stiffeners should be determined using a beam bending analysis of the corrugated profile, assuming that the wall is continuous over the rings and including the consequences of different radial displacements of ring stiffeners that have different sizes. The stresses arising from this bending should be added to those arising from axial compression when checking the buckling resistance under axial compression.

   NOTE: The vertical bending of the sheeting can be analysed by treating it as a continuous beam passing over flexible supports at the ring locations. The stiffness of each support is then determined from the ring stiffness to radial loading.

(3) The ring stiffeners designed to carry the horizontal load should be proportioned in accordance with EN 1993-1-1 and EN 1993-1-3 as appropriate.

5.3.5.3 Buckling under axial compression

(1) The critical buckling stress for the wall should be determined using the provisions of EN 1993-1-3 (cold formed construction), and treating the corrugated sheeting cross-section as a column acting between stiffening rings. The effective length should be taken as not less than the separation of the centroids of adjacent rings.

5.3.5.4 Buckling under external pressure, partial vacuum or wind

(1) The design resistance under external pressure should be assessed in the same manner as for horizontally corrugated silos (see 5.3.4.5), but taking account of the changed orientation of the corrugations as noted in 4.4 (7).
5.3.5.5 Membrane shear

(1) The design resistance under membrane shear should be assessed as for horizontally corrugated silos, see 5.3.4.6.

5.4 Special support conditions for cylindrical walls

5.4.1 Shell with bottom fully supported or resting on a grillage

(1) Where the base of the cylindrical shell is fully supported, the forces and moments in the shell wall may be deemed to be only those induced under axisymmetric actions and patch loads as set out in EN 1991-4.

(2) Where stiffened wall construction is used, the vertical stiffeners should be fully supported by the base and connected to the base ring.

5.4.2 Shell supported by a skirt

(1) If the shell is supported on a skirt (see figure 5.6), the shell may be assumed to be uniformly supported provided that the skirt satisfies one of the two following conditions:
   a) The skirt is itself fully uniformly supported by the foundation;
   b) The thickness of the skirt is not less than 20% greater than the shell, and the ring girder design procedures given in section 8 are used to proportion the skirt and its adjoining flanges.

(2) The skirt should be designed to carry the axial compression in the silo wall without the beneficial effect of internal pressure.

5.4.3 Cylindrical shell wall with engaged columns

(1) If the shell is supported on discrete columns that are engaged into the wall of the cylinder (see figure 5.6b), the effects of the discrete forces from these supports should be included in determining the internal forces in the shell for silos of Consequence Classes 2 and 3.

(2) The length of the engagement of the column should be determined according to 5.4.6.

(3) The length of the rib should be chosen taking account of the limit state of buckling in shear adjacent to the rib, see 5.3.2.6.
5.4.4 Discretely supported cylindrical shell

(1) If the shell is supported on discrete columns or supports, the effects of the discrete forces from these supports should be included in determining the internal forces in the shell, except where the provisions of (2) and (3) permit them to be ignored.

(2) If the shell is analysed using only the membrane theory of shells for axisymmetric loading, the following four criteria should all be satisfied:

a) The radius-to-thickness ratio \( r/t \) should not be more than \( (r/t)_{\text{max}} \).

b) The eccentricity of the support beneath the shell wall should not be more than \( k_1 t \).

c) The cylindrical wall should be rigidly connected to a hopper that has a wall thickness not less than \( k_2 t \) at the transition.

d) The width of each support should be not less than \( k_3 \sqrt{rt} \).

NOTE: The National Annex may choose the values of \( (r/t)_{\text{max}} \), \( k_1 \), \( k_2 \) and \( k_3 \). The values \( (r/t)_{\text{max}} = 400 \), \( k_1 = 2.0 \), \( k_2 = 1.0 \), \( k_3 = 1.0 \) are recommended.

(3) If the shell is analysed using only the membrane theory of shells for axisymmetric loading, one of the following criteria should be met:

a) The upper edge shell boundary condition should be kept circular by structural connection to a roof.

b) The upper edge shell boundary should be kept circular by using a top edge ring stiffener with a flexural rigidity \( EI_z \) for bending in the plane of the circle greater than \( EI_{z,\text{min}} \) given by:

\[
EI_{z,\text{min}} = k_s \frac{E_{zz} t^3}{12}
\]

where \( t \) should be taken as the thickness of the thinnest part of the wall.

NOTE: The National Annex may choose the value of \( k_s \). The value \( k_s = 0.10 \) is recommended.
c) The shell height $L$ should not be less than $L_{s,\text{min}}$, which may be calculated as:

$$L_{s,\text{min}} = k_L r \sqrt{\frac{r}{t}} \cdot \frac{1}{n(n^2 - 1)}$$  \hspace{1cm} (5.83)

where $n$ is the number of supports around the shell circumference.

**NOTE:** The National Annex may choose the value of $k_L$. The value $k_L = 4.0$ is recommended.

(4) If linear shell bending theory or a more precise analysis is used, the effects of locally high stresses above the supports should be included in the verification for the axial compression buckling limit state, as detailed in 5.3.2.4.

(5) The support for the shell should be proportioned to satisfy the provisions of 5.4.5 or 5.4.6 as appropriate.

### 5.4.5 Discretely supported silo with columns beneath the hopper

(1) A silo should be deemed to be supported beneath its hopper if the vertical line above the centroid of the supporting member is more than $t$ inside the middle surface of the cylindrical shell above it.

(2) A silo supported beneath its hopper should satisfy the provisions of section 6 on hopper design.

(3) A silo supported by columns beneath its hopper should be analysed using linear shell bending theory or a more precise analysis. The local bending effects of the supports and the meridional compression that develops in the upper part of the hopper should be included in the verification for both the plastic limit state and the buckling limit state, and these verifications should be carried out using EN 1993-1-6.

### 5.4.6 Local support details and ribs for load introduction in cylindrical walls

#### 5.4.6.1 Local supports beneath the wall of a cylinder

(1) A local support bracket beneath the wall of a cylinder should be proportioned to transmit the design force without localised irreversible deformation to the support or the shell wall.

(2) The support should be proportioned to provide appropriate vertical, circumferential and meridional rotational restraint to the edge of the cylinder.

**NOTE:** Some possible support details are shown in figure 5.7.
Local support at transition ring with engaged column

Possible stiffening arrangement for cylindrical wall with high local support loads

Figure 5.7: Typical details of supports

(3) The length of engagement should be chosen taking account of the limit state of buckling of the shell in shear adjacent to the engaged column, see 5.3.2.6.

(4) Where discrete supports are used without a ring girder, the stiffener above each support should be either:
   a) engaged into the shell as far as the eaves;
   b) engaged by a distance not less than $L_{\text{min}}$, determined from:

$$L_{\text{min}} = 0.4r \sqrt{\frac{r}{t}} \cdot \frac{1}{n(n^2 - 1)}$$  \hspace{1cm} \ldots (5.84)

where $n$ is the number of supports around the shell circumference.

5.4.6.2 Local ribs for load introduction into cylindrical walls

(1) A rib for local load introduction into the wall of a cylinder should be proportioned to transmit the design force without localised irreversible deformation to the support or the shell wall.

(2) The engagement length of the rib should be chosen taking account of the limit state of buckling of the shell in shear adjacent to the rib, see 5.3.2.6.

(3) The design of the rib should take account of the need for rotational restraint of the rib to prevent local radial deformations of the cylinder wall. Where necessary, stiffening rings should be used to prevent radial deformations.

**NOTE:** Possible details for load introduction into the shell using local ribs are shown in figure 5.8.
5.4.7 Anchorage at the base of a silo

(1) The design of the anchorage should take account of the circumferential non-uniformity of the actual actions on the shell wall. Particular attention should be paid to the local high anchorage requirements needed to resist wind action.

**NOTE:** Anchorage forces are usually underestimated if the silo is treated as a cantilever beam under global bending.

(2) The separation between anchorages should not exceed the value derived from consideration of the base ring design, given in 8.5.3.

(3) Unless a more thorough assessment is made using numerical analysis, the anchorage design should have a resistance adequate to sustain the local value of the uplifting force $n_{x,Ed}$ per unit circumference:

$$n_{x,Ed} = \rho_{n,Edw} \left( \frac{L^2}{2r} \right) \left[ C_1 + \sum_{m=2}^{u} m^2 C_m \left\{ 1 - \frac{3}{4} \left( \frac{a_1}{a_2 + a_3} \right) \right\} \right]$$

... (5.85)

$$a_1 = 1 + 10.4 \left( \frac{r}{mL} \right)^2$$

... (5.86)

$$a_2 = 1 + 7.8 \left( \frac{r}{mL} \right)^2$$

... (5.87)

$$a_3 = 3 \frac{r^{3/2}}{L} \left( \frac{L}{r} \right)^{3/2} \left( \frac{1}{m^2 (m^2 - 1)^2} \right)$$

... (5.88)

where:

- $\rho_{n,Edw}$ is the design value of the stagnation point pressure under wind;
- $L$ is the total height of the cylindrical shell wall;
- $t$ is the mean thickness of the cylindrical shell wall;
II is the second moment of area of the ring at the upper edge of the cylinder about its vertical axis (circumferential bending);
\[ C_n \] are the harmonic coefficients of the wind pressure distribution around the circumference
\[ M \] is the highest harmonic in the wind pressure distribution.

NOTE: The values for the harmonic coefficients of wind pressure \( C_n \) relevant to specific conditions may be chosen by the National Annex. The following gives a simple recommendation for Class 1 and 2 silos: \( M = 4 \), \( C_1 = +0.25 \), \( C_2 = +1.0 \), \( C_3 = +0.45 \) and \( C_4 = -0.15 \). For Class 3 silos, the more precise distributions with \( M = 4 \) for isolated silos and \( M = 10 \) for grouped silos given in Annex C are recommended.

5.5 Detailing for openings in cylindrical walls

5.5.1 General
(1) Openings in the wall of the silo should be reinforced by vertical and horizontal stiffeners adjacent to the opening. If any material of the shell wall lies between the opening and the stiffener, it should be ignored in the calculation.

5.5.2 Rectangular openings
(1) The vertical reinforcement around a rectangular opening (see figure 5.9) should be dimensioned so that the cross-sectional area of the stiffeners is not less than the cross-sectional area of the wall that has been lost, but not more than twice this value.
(2) The horizontal reinforcement should be dimensioned so that the cross-sectional area of the stiffeners is not less than the cross-sectional area of the wall that has been lost.
(3) The flexural stiffness of the stiffeners orthogonal to the direction of the membrane stress resultant should be chosen so that the relative displacement \( \delta \) of the shell wall in the direction of the stress resultant on the centreline of the opening and resulting from the presence of the opening is not greater than \( \delta_{\text{max}} \), determined as:
\[
\delta_{\text{max}} = k_d \sqrt{\frac{1}{p}} \cdot d
\]...
(5.89)

where \( d \) is the width of the opening normal to the direction of the stress resultant.

NOTE: The National Annex may choose the value of \( k_d \). The value \( k_d = 0.02 \) is recommended.
(4) The vertical reinforcing stiffeners should extend not less than \( 2\sqrt{\pi} \) above and below the opening.
(5) The shell should be designed to resist local buckling of the wall adjacent to the termination of the stiffeners using the provisions of 5.4.5 and 5.4.6 for local loads.
5.6 Serviceability limit states

5.6.1 Basis

(1) The serviceability limit states for steel silo cylindrical plated walls should be taken as:
   - deformations or deflections that adversely affect the effective use of the structure;
   - deformations, deflections, vibration or oscillation that causes damage to both structural and non-structural elements.

(2) Deformations, deflections and vibrations should be limited to meet the above criteria.

(3) Specific limiting values, appropriate to the intended use, should be agreed between the designer, the client and the relevant authority, taking account of the intended use and the nature of the solids to be stored.

5.6.2 Deflections

(1) The limiting value for global horizontal deflection should be taken as:

\[ w_{\text{max}} = k_d H \]  \hspace{1cm} (5.90)

where \( H \) is the height of the structure measured from the foundation to the roof.

**NOTE:** The National Annex may choose the value of \( k_d \). The value \( k_d = 0.02 \) is recommended.

(2) The limiting value for local radial deflection (departure of cross-section from circular) under wind should be taken as the lesser of:

\[ w_{t,\text{max}} = k_{d3} r \]  \hspace{1cm} (5.91)

\[ w_{t,\text{max}} = k_{d4} t \]  \hspace{1cm} (5.92)

where \( t \) is the local thickness of the thinnest part of the shell wall.

**NOTE:** The National Annex may choose the values of \( k_{d3} \) and \( k_{d4} \). The values \( k_{d3} = 0.05 \) and \( k_{d4} = 20 \) are recommended.
6 Design of conical hoppers

6.1 Basis

6.1.1 General

(1) Conical hoppers should be so proportioned that the basic design requirements for ultimate limit states given in section 2 are satisfied.

(2) The safety assessment of the conical shell should be conducted using the provisions of EN 1993-1-6.

6.1.2 Hopper wall design

(1) The conical wall of the hopper should be checked for:
- resistance to rupture under internal pressure and wall friction;
- resistance to local yielding in bending at the transition;
- resistance to fatigue failure;
- resistance of joints (connections);
- resistance to buckling under transverse loads from feeders and attachments;
- local effects.

(2) The shell wall should satisfy the provisions of EN 1993-1-6, except where 6.3 to 6.5 provide conditions that are deemed to satisfy the provisions of that standard.

(3) The rules given in 6.3 to 6.5 may be used for hoppers with hopper half angles in the range $0^\circ < \beta < 70^\circ$.

(4) For hoppers in Consequence Class 1, the cyclic plasticity and fatigue limit states may be ignored, provided that both the following two conditions are met:
   a) The design for the rupture at the transition junction should be carried out using an enhanced partial factor of $\lambda_{M0} = \lambda_{M0e}$.
   b) No local meridional stiffeners or supports are attached to the hopper wall near the transition junction.

   NOTE: The National Annex may choose the value of $\lambda_{M0e}$. The value $\lambda_{M0e} = 1.4$ is recommended.

6.2 Distinctions between hopper shell forms

(1) A hopper wall constructed from flat rolled steel sheet should be termed 'isotropic'.

(2) A hopper wall with stiffeners attached to the outside should be termed 'externally stiffened'.

(3) A hopper with more than one discharge orifice should be termed 'multiple outlet'.

(4) A hopper which forms part of a silo supported on discrete column or bracket supports should be termed 'discretely supported', even though the discrete supports are not directly beneath the hopper.
6.3 Resistance of conical hoppers

6.3.1 General

(1) The conical hopper should satisfy the provisions of EN 1993-1-6. Alternatively, these may be deemed to be satisfied using the assessments of the design resistance given in 6.3.

(2) Special attention should be paid to the possibility that different parts of the hopper can be critically loaded under the pressure patterns of either filling or discharge actions.

(3) The stress resultants arising in the body of the hopper may generally be found using the membrane theory of shells.

NOTE: Additional information relating to the pressure patterns which may occur and the membrane theory stress resultants in the hopper body is given in Annex B.

Figure 6.1: Hopper shell segment

6.3.2 Isotropic unstiffened welded or bolted hoppers

6.3.2.1 General

(1) A conical hopper should be treated as a shell structure, recognising the coupling of meridional and circumferential actions in supporting loads.
6.3.2.2 Plastic mechanism or rupture in the hopper body

(1) The design against rupture should recognise that the hopper can be subject to different patterns and changing patterns of pressures on the wall. Because failure by rupture can easily propagate and is generally not ductile, every point in the hopper should be able to resist the most severe design condition.

(2) Welded or bolted joints running down the meridian within the conical hopper should be proportioned at each point to sustain the worst membrane forces arising from either the filling or the discharge pressure distribution.

(3) Welded or bolted joints running around the hopper circumference should be proportioned to sustain the maximum total weight of solids that can be applied below that point.

NOTE: This is generally defined by the filling pressure distribution; see EN 1991-4.

6.3.2.3 Rupture at the transition junction

(1) The circumferential joint between the hopper and the transition junction, see Figure 6.2, should be designed to carry the maximum total meridional load that the hopper can be required to support, allowing for possible unavoidable non-uniformities.

![Critical joint](Image)

Figure 6.2: Hopper transition joint: potential for rupture

(2) Where the only loading under consideration is gravity and flow loading from the stored solid, the meridional force per unit circumference $n_{ph,Ed,s}$ caused by the symmetrical pressures defined in EN 1991-4 that must be transmitted through the transition joint should be evaluated using global equilibrium. The design value of the local meridional force per unit circumference $n_{ph,Ed}$, allowing for the possible non-uniformity of the loading, should then be obtained as

$$n_{ph,Ed} = k_{asym} n_{ph,Ed,s}$$

where:

- $n_{ph,Ed,s}$ is the design value of the meridional membrane force per unit circumference at the top of the hopper obtained assuming the hopper loads are entirely symmetrical;
- $k_{asym}$ is the unsymmetrical stress augmentation factor.
NOTE: Expressions for $n_{q_{h\text{-}Ed}}$ may be found in Annex B. The National Annex may choose the value of $g_{\text{asym}}$. The value $g_{\text{asym}} = 1.2$ is recommended.

(3) For silos in Consequence Class 2, an elastic bending analysis should be made of the hopper where other loads from discrete supports, feeders, attached members, non-uniform hopper pressures etc. are involved. This analysis should determine the maximum local value of the meridional force per unit circumference to be transmitted through the hopper to transition junction joint.

(4) The design resistance of the hopper at the transition joint $n_{q_{h\text{-Rd}}}$ should be taken as:

$$n_{q_{h\text{-Rd}}} = k_r f_u / K_{M2} \quad \ldots (6.2)$$

where $f_u$ is the tensile strength.

NOTE: The National Annex may choose the value of $k_r$. The value $k_r = 0.90$ is recommended.

6.3.2.4 Plastic mechanism at thickness changes or at the transition

(1) The plastic mechanism resistance of the hopper should be evaluated in terms of the local value of meridional membrane stress resultant $n_q$ at the upper edge of the cone or at a change of plate thickness.

(2) The design resistance $n_{q_{Rd}}$ should be determined from:

$$n_{q_{Rd}} = \left( \frac{rt}{r-2.4} \frac{rt}{\cos \beta \cdot \sin \beta} \right) \left( \frac{0.91 \mu + 0.27}{\mu + 0.15} \right) / K_{M0} \quad \ldots (6.3)$$

where:

- $t$ is the local wall thickness;
- $r$ is the radius at the top of the plastic mechanism (hopper top or change of plate thickness);
- $\beta$ is the hopper half angle, see figure 6.1;
- $\mu$ is the wall friction coefficient for the hopper.
(3) At each critical point in the structure, the design stresses should satisfy the condition:

\[ n_{\phi,Ed} \leq n_{\phi,Rd} \quad \ldots \ (6.4) \]

**Figure 6.3: Plastic collapse of conical hopper**

### 6.3.2.5 Local flexure at the transition

(1) To avoid cyclic plasticity and fatigue failures, the hopper should be designed to resist the severe local flexure at the top of the hopper that arises from both compatibility and equilibrium effects.

(2) This requirement may be ignored for silos of Consequence Class I.

(3) In the absence of a finite element analysis of the structure, the value of the local bending stress at the top of the hopper should be assessed using the following procedure.

(4) The effective radial force \( F_{c,Ed} \) and moment \( M_{c,Ed} \) acting on the transition ring should be determined from:

\[ F_{c,Ed} = n_{\phi,Ed} \sin \beta - F_h - F_c \quad \ldots \ (6.5) \]

\[ M_{c,Ed} = F_c x_c - F_h x_h \quad \ldots \ (6.6) \]

with:

\[ F_c = 2 x_c \ p_{nc} \quad \ldots \ (6.7) \]

\[ F_h = 2 x_h (0.85 - 0.15 \mu \cot \beta) \ p_{nh} \quad \ldots \ (6.8) \]

\[ x_c = 0.39 \sqrt{r_c} \quad \ldots \ (6.9) \]

\[ x_h = 0.39 \sqrt{\frac{r_h}{\cos \beta}} \quad \ldots \ (6.10) \]

where (see figure 8.4):
The local bending stress $\sigma_{\text{hop},Ed}$ at the top of the hopper should be determined from:

$$
\sigma_{\text{hop},Ed} = \left( \frac{6}{\Delta} \right) \left\{ (a_2 - 2a_3\eta)M_{c,Ed} - \rho(a_3 - a_2\eta)F_{c,Ed} \right\} - \left( \frac{6}{t_h^2} \right) F_h x_h

(6.11)
$$

with:

$$
\Delta = 2a_3 - a_2^2

(6.12)
$$

$$
\rho = 0.78 \sqrt{r}

(6.13)
$$

$$
\eta = \sqrt{t_h \cos \beta}

(6.14)
$$

$$
a_1 = t_h^{3/2} + t_c^{3/2} + t_h^{3/2} \frac{A_{ep}}{\rho}

(6.15)
$$

$$
a_2 = t_c^2 - t_h^2 + t_h^2

(6.16)
$$

$$
a_3 = t_h^{5/2} + t_c^{5/2} + t_h^{5/2} \cos \beta

(6.17)
$$

where:

$t_h$ is the hopper local wall thickness;

$t_c$ is the local wall thickness of the cylinder at the transition junction;

$r$ is the radius of the transition junction (top of the hopper);

$A_{ep}$ is the cross-sectional area of the ring at the transition junction (without any effective contributions from the adjacent shell segments);

$\beta$ is the hopper apex half angle;

$\rho$ is the wall friction coefficient for the hopper.

### 6.3.2.6 Hoppers that are part of a silo resting on discrete supports

(1) If the silo is supported on discrete supports or columns, the relative stiffness of the transition ring girder, cylinder wall and hopper should be taken into account when assessing the non-uniformity of the meridional membrane stresses in the hopper.

(2) This requirement may be ignored for silos of Consequence Class 1.
The hopper should be designed to sustain the highest local value of meridional tension at the hopper top (adjacent to a support) according to 6.3.2.3 and 6.3.2.4.

6.3.2.7 Buckling in hoppers

(1) This criterion may be ignored for silos of Consequence Class 1.

(2) The hopper should be assessed for its resistance to buckling failure as a consequence of horizontal actions from feeders or attached structures, or as a result of unsymmetrical vertical actions.

(3) The design buckling resistance \( n_{\text{qh,Rd}} \) at the top of the hopper should be determined from:

\[
 n_{\text{qh,Rd}} = 0.6 \alpha_{\text{sh}} E \left( \frac{r^2}{h} \right) \cos \beta / \gamma_{M1} 
\]

where:
- \( \alpha_{\text{sh}} \) is the elastic buckling imperfection sensitivity factor;
- \( t_h \) is the hopper local wall thickness;
- \( r \) is the radius of the transition junction (top of the hopper).

and \( \gamma_{M1} \) is given in 2.9.2, but \( n_{\text{q,Rd}} \) should not be taken as greater than \( n_{\text{q,Rd}} = t_h \frac{f_y}{\gamma_{M1}} \).

NOTE: The National Annex may choose the value of \( \alpha_{\text{sh}} \). The value \( \alpha_{\text{sh}} = 0.10 \) is recommended.

(4) The meridional force at the top of the hopper should satisfy the condition:

\[
 n_{\text{qh,Ed}} \leq n_{\text{qh,Rd}} 
\]

6.4 Considerations for special hopper structures

6.4.1 Supporting structures

(1) The effect of discrete supports beneath the silo should be treated as set out in 5.4. The supporting structures themselves should be designed to EN 1993-1-1, with the boundary between the silo and supporting structure as defined in 1.1 (4).

6.4.2 Column supported hopper

(1) If the hopper body itself is supported on discrete supports or columns that do not reach the hopper top edge, the hopper structure should be analysed using the bending theory of shells, see EN 1993-1-6.

(2) Adequate provision should be made to distribute the support forces into the hopper.

(3) The joints in the hopper should be designed for the highest local value of stress resultants to be transmitted through them.

(4) The hopper should be assessed for resistance to buckling failure in zones where compressive membrane stresses develop, see EN 1993-1-6.
6.4.3 Unsymmetrical hopper

(1) If the axis of the hopper is not vertical, but inclined at an angle $\omega$ to the vertical (Fig. 6.4), the increased meridional stresses on the steep side associated with this geometry should be evaluated, and appropriate provision made to provide an adequate local meridional resistance.

6.4.4 Stiffened cones

(1) The stringer stiffeners should be adequately anchored at the top of the hopper.

(2) If the hopper cone is stiffened with meridional stiffeners, the effects of compatibility between the wall plate and stringers should be included. The effect of the circumferential tension in the hopper wall should be included in the assessment of the forces in the stringer stiffeners and the hopper wall plate, as affected by the Poisson effect.

(3) The hopper plate joints should be proportioned to resist the increased tension arising from compatibility.

(4) The connection between the stringer and hopper plate should be proportioned for the interaction forces between them.

![Figure 6.4: Unsymmetrical hopper with engaged columns in cylinder](image)

6.4.5 Multi-segment cones

(1) If a hopper cone is composed of several segments with different slopes, the appropriate bulk solids actions on each segment should be evaluated and included in the structural design.

(2) The local circumferential tensions or compressions at changes in hopper slope should be evaluated, and adequate resistance provided to support them.

(3) The potential for severe local wear at such changes in hopper slope should be included in the design.

6.5 Serviceability limit states

6.5.1 Basis

(1) If serviceability criteria are deemed necessary, specific limiting values for hoppers should be agreed between the designer and the client.
6.5.2 Vibration

(1) Provision should be made to ensure that the hopper is not subject to excessive vibration during operation.
7 Design of circular conical roof structures

7.1 Basis

(1) The design of roof structures should take into consideration permanent, transient, imposed, wind, snow, accidental and partial vacuum loads.

(2) The design should also take account of the possibility of upward forces on the roof due to accidental overfilling or unexpected fluidisation of stored solids.

7.2 Distinctions between roof structural forms

7.2.1 Terminology

(1) A conical shell roof formed from rolled plates and without supporting beams or rings should be termed a 'shell roof' or an 'unsupported roof'.

(2) A conical roof in which sheeting is supported on beams or a grillage should be termed a 'framed roof' or a 'supported roof'.

7.3 Resistance of circular conical silo roofs

7.3.1 Shell or unsupported roofs

(1) Shell roofs should be designed according to the requirements of EN 1993-1-6, but the following provisions may be deemed to satisfy them for conical roofs with a diameter not greater than 5m and a roof inclination to the horizontal $\phi$ not greater than 40°.

(2) The calculated surface von Mises equivalent stresses due to combined bending and membrane action should everywhere be limited to the value:

$$ f_{e,Rd} = f_y / \gamma_{M0} $$

... (7.1)

where $\gamma_{M0}$ is obtained from 2.9.2.

(3) The critical buckling external pressure $p_{n,Rec}$ for an isotropic conical roof should be calculated as:

$$ p_{n,Rec} = 2.65E \left( \frac{t \cos \phi}{r} \right)^{2.43} \cdot (\tan \phi)^{1.6} $$

... (7.2)

where:

- $r$ is the outer radius of the roof;
- $t$ is the smallest shell plate thickness;
- $\phi$ is the slope of the cone to the horizontal.

(4) The design buckling external pressure should be determined as:

$$ p_{n,Rd} = \alpha_p p_{n,Rec} / \gamma_{M1} $$

... (7.3)

in which $\gamma_{M1}$ is obtained from 2.9.2.

**NOTE:** The National Annex may choose the value of $\alpha_p$. The value $\alpha_p = 0.20$ is recommended.
(5) The design peak external pressure on the roof arising from the actions defined in 7.1 should satisfy the condition:

\[ p_{n,Ed} \leq p_{n,Rd} \]  \hspace{2cm} \ldots (7.4)

7.3.2 Framed or supported roofs

(1) Framed or supported roofs, where the roof sheeting is supported on beams or a grillage should be designed according to the provisions of EN 1993-4-2 (Tanks).

7.3.3 Eaves junction (roof to shell junction)

(1) The roof to shell junction, and the ring stiffener at this junction should be designed according to the provisions of EN 1993-4-2 (Tanks).
8 Design of transition junctions and supporting ring girders

8.1 Basis

8.1.1 General

(1) A steel transition ring or ring girder should be so proportioned that the basic design requirements for the ultimate limit state given in section 2 are satisfied.

(2) The safety assessment of the ring should be carried out using the provisions of EN 1993-1-6, except where the provisions of this Standard are deemed to satisfy them.

(3) For silos in Consequence Class I, the cyclic plasticity and fatigue limit states may be ignored, provided that the following conditions are met.

8.1.2 Ring design

(1) The ring or ring girder should be checked for:
   - resistance to plastic limit under circumferential compression;
   - resistance to buckling under circumferential compression;
   - resistance to local yielding under tension or compression stresses;
   - resistance to local failure above supports;
   - resistance to torsion;
   - resistance of joints (connections).

(2) The ring girder should satisfy the provisions of EN 1993-1-6, except where 8.2 to 8.5 provide conditions that are deemed to satisfy the provisions of that standard.

(3) For silos in Consequence Class I, the cyclic plasticity and fatigue limit states may be ignored.

8.1.3 Terminology

(1) A ring whose purpose is only to provide resistance to radial components of forces from the hopper should be termed a ‘transition ring’.

(2) A ring whose purpose is to provide redistribution of vertical forces between different components (e.g. the cylinder wall and discrete supports), should be termed a ‘ring girder’.

(3) The point of intersection between the middle surface of the hopper plate and the middle surface of the cylindrical shell wall at the transition junction, termed the ‘joint centre’, should be used as the reference point in limit state verifications.

(4) A silo with no identified ring at the transition (see figure 8.1) has an effective ring formed from adjacent shell segments and should be termed a ‘natural ring’.

(5) An annular plate placed at the transition junction should be termed an ‘annular plate ring’, see figure 8.1.

(6) A hot rolled steel section, used as a ring stiffener at the transition should be termed a ‘rolled section ring’.

(7) A rolled steel section rolled around the silo circumference and used to support the shell beneath the transition should be termed a ‘rolled ring girder’.
A section built up from steel plates with cylindrical and annular plate forms should be termed a 'fabricated ring girder', see figure 8.1.

### 8.1.4 Modelling of the junction

1. In hand calculations, the junction should be represented by cylindrical and conical shell segments and annular plates only.

2. Where the silo is uniformly supported, the circumferential stresses in the annular plates of the junction may be assumed to be uniform in each plate.

3. Where the silo is supported on discrete supports or columns, the circumferential stresses in the junction plates should be taken to vary radially in each plate as a consequence of warping stresses.

![Figure 8.1: Example ring forms](image)

#### 8.1.5 Limitations on ring placement

1. The vertical eccentricity of any annular plate or ring from the transition joint centre should not be greater than \( 0.2 \sqrt{t} \), where \( t \) is the thickness of the cylinder plate, unless a shell bending calculation according to EN 1993-1-6 is carried out to check the effect of the eccentricity.

   **NOTE:** This rule arises from the ineffectiveness of rings placed further than this from the junction, see figure 8.2.

2. The simplified rules in 8.2 apply only where this requirement is met.

### 8.2 Analysis of the junction

#### 8.2.1 General

1. For silos in Consequence Class 1, the transition junction may be analysed using simple expressions and loadings from adjacent shell segments derived from membrane theory.

2. Where a computer calculation of the transition junction is performed, it should satisfy the requirements of EN 1993-1-6.

3. Where a computer calculation is not used and the silo is uniformly supported, the analysis of the junction may be undertaken using 8.2.2.

4. Where a computer calculation is not used and the silo is supported on discrete supports or columns, the analysis of the junction should be undertaken using 8.2.3.
8.2.2 Uniformly supported transition junctions

(1) The effective section of the transition junction should be evaluated as follows: the shell segments meeting at the joint centre should be separated into those above (Group A) and those below (Group B), see figure 8.3 (a). All annular plate segments at the level of the joint centre should be initially ignored. Where a vertical leg is attached to the annular plate at a different radial coordinate from the joint centre, it should be treated as a shell segment in the same manner as the others, see figure 8.3.

(2) The equivalent thickness $t_{eqA}$ and $t_{eqB}$ of each group should be determined from:
(3) The ratio $\alpha$ of the thinner to the thicker equivalent plate group should be determined from:

$$\alpha = \frac{(t_{eq})_{thinner}}{(t_{eq})_{thicker}}$$  \hspace{1cm} (8.3)

with:

$$(t_{eq})_{thinner} = \min(t_{eqA}, t_{eqB})$$  \hspace{1cm} (8.4)

$$(t_{eq})_{thicker} = \max(t_{eqA}, t_{eqB})$$  \hspace{1cm} (8.5)

(4) For the thinner of these two groups, the effective length of each shell segment should be determined from:

$$\ell_{e1} = 0.778 \sqrt{\frac{rt}{\cos \beta}}$$  \hspace{1cm} (8.6)

where $\beta$ is the angle between the shell centreline and the silo axis (cone apex half angle) for that plate. The effective cross-sectional area of each shell segment should be determined from:

$$A_{e1} = \ell_{e1} t$$  \hspace{1cm} (8.7)

For the thicker of these two groups, the effective length of each shell segment should be determined from:

$$\ell_{e2} = 0.389 \left[ 1 + 3\alpha^2 - 2\alpha \right] \sqrt{\frac{rt}{\cos \beta}}$$  \hspace{1cm} (8.8)

For this group, the effective cross-sectional area of each shell segment should be determined from:

$$A_{e2} = \ell_{e2} t$$  \hspace{1cm} (8.9)
(5) The effective cross-sectional area $A_{ep}$ of the annular plate joining into the junction at the joint centre should be determined from:

$$A_{ep} = \frac{bt_p}{1 + 0.8 \frac{b}{r}}$$

... (8.10)

where:

$r$ is the radius of the silo cylinder wall;

$b$ is the radial width of the annular plate ring;

$t_p$ is the thickness of the annular plate ring.

(6) The total effective area $A_{et}$ of the ring in developing circumferential compression should be determined from:

$$A_{et} = A_{ep} + \sum_{i=1}^{\text{all segments}} A_{ei}$$

... (8.11)

(7) Where the junction consists only of a cylinder, skirt and hopper (see figure 8.4), the total effective area of the ring $A_{et}$ may be alternatively found from:

$$A_{et} = A_{ep} + 0.778 \sqrt{r} \left[ t_c^{3/2} + \psi \left( t_b^{3/2} \cos \beta + t_s^{3/2} \right) \right]$$

... (8.12)

with:

$$\psi = 0.5 \left( 1 + 3\alpha^2 - 2\alpha^3 \right)$$

... (8.13)

$$\alpha = \frac{t_c}{\sqrt{t_s^2 + t_h^2}}$$

... (8.14)

where:

$r$ is the radius of the silo cylinder wall;

$t_c$ is the thickness of the cylinder;

$t_s$ is the thickness of the skirt;

$t_h$ is the thickness of the hopper;

$A_{ep}$ is the effective area of the annular plate ring.
(8) Where sections of more complex geometry are used at the transition junction, only ring plate segments meeting the condition of 8.1.5 (1) should be deemed to be effective in the evaluation of the junction.

(9) The design value of the effective circumferential compressive force $N_{\theta,Ed}$ developed in the junction should be determined from:

$$N_{\theta,Ed} = n_{\phih,Ed} r \sin \beta - p_{nc} r \ell_{cc} - p_{nh} (\cos \beta - \mu \sin \beta) r \ell_{eh}$$

where (see figure 8.5):

- $r$ is the radius of the silo cylinder wall;
- $\beta$ is the half angle of the hopper (at the top);
- $\ell_{cc}$ is the effective length of the cylinder segment above the transition (see (4));
- $\ell_{eh}$ is the effective length of the hopper segment (see (4));
- $n_{\phih,Ed}$ is the design value of the meridional tension per unit circumference at the top of the hopper;
- $p_{nc}$ is the mean local pressure on the effective length of the cylinder segment;
- $p_{nh}$ is the mean pressure on the effective length of the hopper segment;
- $\mu$ is the hopper wall friction coefficient.
Figure 8.5: Local pressures and membrane stress resultant loadings on the transition ring

(10) The maximum design compressive stress $\sigma_{u0,Ed}$ for the uniformly supported junction should be determined from:

$$\sigma_{u0,Ed} = \frac{N_{u0,Ed}}{\eta A_{ct}}$$  \hspace{1cm} \text{(8.16)}

with:

$$\eta = 1 + 0.3 \frac{b}{r}$$  \hspace{1cm} \text{(8.17)}

where:
- $N_{u0,Ed}$ is the effective circumferential compressive force, see (9);
- $A_{ct}$ is the total effective area of the ring, see (7);
- $r$ is the radius of the silo cylinder wall;
- $b$ is the width of the annular plate.

### 8.2.3 Transition junction ring girder

(1) For silos in Consequence Class 3, a numerical analysis of the structure should be carried out, that models all plate elements as shell segments, and does not assume prismatic beam action in any curved element. The analysis should take account of the finite width of the discrete supports.

(2) For silos in other Consequence Classes, the bending moments and torques within the ring girder should be calculated, accounting for the eccentricities of loading and support from the ring girder centroid.

(3) The total circumferential compressive thrust developed in the girder should be assumed invariant around the circumference and determined from:

$$N_{6,Ed} = n_{u0,Ed} r_c \sin \beta - p_{nc} r_c \ell_{ec} - p_{nh} (\cos \beta - \mu \sin \beta) r_c \ell_{eh}$$  \hspace{1cm} \text{(8.18)}

where (see figure 8.5):
$r_c$ is the radius of the silo cylinder wall;  
$\beta$ is the half angle of the hopper (at the top);  
$\ell_{ec}$ is the effective length of the cylinder segment above the transition (see 8.2.2 (4));  
$\ell_{ch}$ is the effective length of the hopper segment (see 8.2.2 (4));  
$n_{\phi h, Ed}$ is the design value of the meridional tension per unit circumference at the top of the hopper;  
$p_{nc}$ is the mean local pressure on the effective length of the cylinder segment;  
$p_{nh}$ is the mean pressure on the effective length of the hopper segment;  
$\mu$ is the hopper wall friction coefficient.

(4) The variation with circumferential coordinate $\theta$ of the design bending moment $M_{r, Ed}$ about the horizontal (radial) axis (sagging positive) and the design torsional moment $T_{0, Ed}$ in the ring girder should be taken as:

$$ M_{r, Ed} = n_{v, Ed} (r_g - e_i) [(r_g - e_i) \theta_o (\sin \theta + \cot \theta_o \cos \theta) - r_g + e_i] + n_{r, Ed} e_s (r_g - e_i) \ldots \ldots (8.19a) $$

$$ T_{0, Ed} = n_{v, Ed} (r_g - e_i) [(r_g - e_i) \theta_o (\cot \theta_o \sin \theta - \cos \theta) + r_g (\theta_o - \theta)] \ldots \ldots (8.19b) $$

with:

$$ \theta_o = \frac{\pi}{j} \ldots \ldots (8.20) $$

$$ n_{v, Ed} = n_{xc, Ed} + n_{\phi h, Ed} \cos \beta \ldots \ldots (8.21a) $$

$$ n_{r, Ed} = n_{\phi h, Ed} \sin \beta \ldots \ldots (8.21b) $$

where (see figure 8.6):

$\theta$ is the circumferential coordinate (in radians) measured from an origin at one support;  
$\theta_o$ is the circumferential angle in radians subtended by the half span of the ring girder;  
j is the number of equally spaced discrete supports;  
$r_g$ is the radius of the ring girder centroid;  
e$_i$ is the radial eccentricity of the cylinder from the ring girder centroid (positive where the centroid is at a larger radius);  
e$_s$ is the radial eccentricity of the support from the ring girder centroid (positive where the centroid is at a larger radius);  
e$_v$ is the vertical eccentricity of the joint centre from the ring girder centroid (positive where the centroid lies below the joint centre);  
n$_{xc, Ed}$ is the design value of compressive membrane stress resultant at the base of the cylinder;  
n$_{\phi h, Ed}$ is the design value of tensile membrane stress resultant at the top of the hopper.

(5) The peak values of the design bending moment about the radial axis that occur over the support $M_{rs, Ed}$ and at midspan $M_{rn, Ed}$ should be determined from:

$$ M_{rs, Ed} = n_{v, Ed} (r_g - e_i) [(r_g - e_i) \theta_o \cot \theta_o - r_g + e_i] + n_{r, Ed} e_s (r_g - e_i) \ldots \ldots (8.22) $$
(6) Where an open section ring girder is used, the torque should be assumed to be resisted entirely by warping, unless a more precise analysis is used. Where warping resists the torques, the peak design values of flange moment about a vertical axis in each flange should be taken as given by $M_{f_{0}, Ed}$ at the support and $M_{f_{m}, Ed}$ at midspan, obtained as follows:

$$M_{f_{0}, Ed} = n_{r, Ed} \frac{r_{e}(r_{e} - e_{r})}{h} \left[ (r_{e} - e_{r})(1 - \theta_{0} \cot \theta_{0}) - \frac{r_{e} \theta_{0}^{2}}{3} \right]$$

... (8.24)

$$M_{f_{m}, Ed} = n_{r, Ed} \frac{r_{e}(r_{e} - e_{r})}{h} \left[ (r_{e} - e_{r})(1 - \theta_{0} \cot \theta_{0}) + \frac{r_{e} \theta_{0}^{2}}{6} \right]$$

... (8.25)

where $h$ is the vertical separation between the flanges of the ring girder.

![Diagram of ring girder showing eccentricities](image)

**Figure 8.6: Eccentricities of vertical loads at a ring girder**

(7) The circumferential membrane stresses $\sigma_{0, Ed}$ that develop in each flange of the ring girder should be determined from the thrust $N_{0, Ed}$, radial axis moment $M_{r, Ed}$ and warping flange moments $M_{w, Ed}$ using engineering bending and warping theory and adopting the stress resultants defined in (3) to (6).

(8) The largest value of the circumferential membrane stress $\sigma_{0, Ed}$ (whether tensile or compressive) that develops in either flange of the ring girder at any position around the circumference should be determined as $\sigma_{0, Ed}$.

(9) The largest compressive value of the circumferential membrane stress $\sigma_{0, Ed}$ that develops in either flange of the ring girder at any position around the circumference should be determined as $\sigma_{0, Ed}$.
8.3 Structural resistances

8.3.1 General

(1) The transition junction should satisfy the provisions of EN 1993-1-6, but these may be met using the following assessments of the design resistance.

8.3.2 Resistance to plastic limit state

8.3.2.1 General

(1) The design value of the resistance should be determined using the provisions of EN 1993-1-6. The following resistance assessments may be used instead as a simple safe approximation to those provisions.

8.3.2.2 Resistance based on elastic evaluation

(1) The design value of the resistance should be determined at the most highly stressed point in the junction.

(2) The design value of the resistance of the plastic limit state should be determined using:

\[ f_{p,Rd} = f_y / \gamma_{M0} \]  

\[ \text{.. (8.26)} \]

8.3.2.3 Resistance based on plastic evaluation

(1) The design value of the resistance should be determined in terms of the attainable tensile membrane stress resultant \( n_{\phi,Rd} \) in the hopper at the junction.

(2) The design value of the resistance at the plastic limit state \( n_{\phi,Rd} \) should be determined using:

\[ n_{\phi,Rd} = \frac{1}{\sin \beta} \left( f_y + \sum p_{\text{w}} (\cos \beta - \mu \sin \beta) l_{\text{ch}} \right) \]  

\[ \text{.. (8.27)} \]

with:

\[ \alpha = \sqrt{\frac{t_z^2}{t_s^2 + t_b^2}} \]  

\[ \text{.. (8.28)} \]

\[ \psi = 0.7 + 0.6 \alpha^2 - 0.3 \alpha^3 \]  

\[ \text{.. (8.29)} \]

- for the cylinder \( \xi_{oc} = 0.975 \sqrt{r t_c} \)
- for the skirt \( \xi_{os} = 0.975 \psi \sqrt{r t_s} \)
- for the conical hopper segment \( \xi_{ch} = 0.975 \psi \sqrt{r t_b / \cos \beta} \)

where (see figure 8.5):

\( r \) is the radius of the silo cylinder wall;

\( t_c \) is the thickness of the cylinder;
ts is the thickness of the skirt;
th is the thickness of the hopper;
Ap is the cross-sectional area of the ring;
β is the half angle of the hopper (at the top);
ℓc is the plastic effective length of the cylinder segment above the transition;
ℓsh is the plastic effective length of the hopper segment;
ℓs is the plastic effective length of the skirt segment below the transition;
nqh,Rd is the meridional membrane resistance per unit circumference at the top of the hopper;

8.3.3 Resistance to in-plane buckling

(1) The design value of the resistance should be determined using the provisions of EN 1993-1-6. The following resistance assessment may be used instead as a simple safe approximation to those provisions.

(2) The design value of the resistance should be assessed using the point in the junction where the highest compressive circumferential membrane stress occurs.

(3) The design value of the resistance against in-plane buckling \( \sigma_{p,Rd} \) should be determined using:

\[
\sigma_{p,Rd} = \frac{4EL_z}{\gamma_m} \frac{1}{A_{et}r_g^2} \frac{1}{\gamma_{M1}}
\]

where:

\( E \)

is the flexural rigidity of the ring effective cross-section (see figure 8.3) about its vertical axis;

\( A_{et} \)

is the effective cross-sectional area of the ring, given by 8.2.2;

\( r_g \)

is the radius of the centroid of the ring effective cross-section.

(4) The above resistance assessment and the associated verification against in-plane buckling of 8.4 may be omitted when the cone half angle \( \beta \) is greater than \( \beta_{lim} \).

NOTE: The National Annex may choose the value of \( \beta_{lim} \). The value \( \beta_{lim} = 20° \) is recommended.

8.3.4 Resistance to out-of-plane buckling and local shell buckling near the junction

8.3.4.1 General

(1) The design value of the resistance should be determined using the provisions of EN 1993-1-6. The following resistance assessments may be used instead as a simple safe approximation to those provisions.
8.3.4.2 Local shell buckling near the junction

(1) For junctions in which there is either no ring at the transition (simple cone to cylinder junction), or the transition is ring stiffened, the design value of the resistance $\sigma_{op,Rd}$ against shell buckling of the wall adjacent to the junction should be determined using:

$$
\sigma_{op,Rd} = \frac{1}{\gamma_{ml}} \cdot 4.1(\cos \beta)^{0.4} \cdot \left( \frac{t}{r_s} \right)^{1.5} \cdot \left( \frac{E_t r_s}{A_{eq}} \right)
$$

... (8.31)

with:

$$
r_s = \frac{r}{\cos \beta}
$$

for the conical hopper wall

where:

$r$ is the radius of the silo cylinder wall;

$\beta$ is the hopper apex half angle;

$t$ is the thickness of the relevant shell segment;

$A_{eq}$ is the effective cross-sectional area of the ring, given by 8.2.2;

$r_s$ is the radius of the centroid of the ring effective cross-section.

8.3.4.3 Annular plate transition junction

(1) For junctions in which the ring at the transition is in the form of an annular plate, the design value of the resistance against out-of-plane buckling $\sigma_{op,Rd}$ should be determined using:

$$
\sigma_{op,Rd} = kE \left( \frac{t}{b} \right)^2 \cdot \frac{1}{\gamma_{ml}}
$$

... (8.32)

with:

$$
k = \frac{\eta_k k_s + \eta \eta_s}{\eta_t + \eta_s}
$$

... (8.33)

$$
k = 0.385 + 0.452 \sqrt{\frac{b}{r}}
$$

... (8.34)

$$
k = 1.154 + 0.56 \frac{b}{r}
$$

... (8.35)

$$
\eta_s = 0.43 + 0.1 \left( \frac{r}{20b} \right)^2
$$

... (8.36)
where:

- \( r \) is the radius of the silo cylinder wall;
- \( t_c \) is the thickness of the cylinder;
- \( t_s \) is the thickness of the skirt;
- \( t_h \) is the thickness of the hopper;
- \( t_p \) is the thickness of the annular plate ring;
- \( b \) is the width of the annular plate ring;
- \( k_c \) is the plate buckling coefficient for a ring with clamped inner edge;
- \( k_s \) is the plate buckling coefficient for a ring with simply supported inner edge;
- \( \gamma_{M1} \) is the partial factor, see 2.9.2.

### 8.3.4.4 T section transition junction

1. The following assessment should be used where the transition junction ring consists of an annular plate of width \( b_p \) with a symmetrically placed vertical stiffening flange of height \( b_f \) at its outer edge, forming a T section ring with the base of the T at the joint centre.

2. The design value of the resistance against out-of-plane buckling \( \sigma_{op,Rd} \) of a T-section ring beam should be determined on the basis of the maximum compressive value of the circumferential membrane stress on the inner edge of the principal annular plate of the ring. The design value of the resistance should be determined from:

\[
\sigma_{op,Rd} = \frac{\eta_{c} \sigma_{s} + \eta_{s} \sigma_{c}}{\eta_{c} + \eta_{s} \gamma_{M1}} \cdot \frac{1}{\gamma_{M1}}
\]

where:

\[
\eta_{c} = 0.385 + \left( \frac{r}{175b_p} \right)^2
\]

\[
\eta_{c} = 0.5 \left( \frac{t_c}{t_p} \right)^{3/2} + \left( \frac{t_s}{t_p} \right)^{3/2} + \left( \frac{t_h}{t_p} \right)^{3/2}
\]

\[
\sigma_{s} = \frac{EI_{f}}{At_{p}^{3}} \left( 0.2 \frac{b_p}{r} + \frac{G\ell_{i}}{E} + 2 \sqrt{\frac{G\ell_{i}b_p}{EI_{f}r}} \right)
\]

\[
\sigma_{c} = \frac{E}{b_p} \left( \frac{t_p}{b_p} \right)^{3/2} \cdot \frac{(1+5\rho)(1+32\rho-16\rho^2)}{64 \left( 1+\frac{b_t}{b_p} \right)^2}
\]
where:

- \( r \) is the radius of the silo cylinder wall;
- \( r_t \) is the thickness of the cylinder;
- \( r_s \) is the thickness of the skirt;
- \( r_h \) is the thickness of the hopper.
- \( t_p \) is the thickness of the annular plate ring;
- \( t_f \) is the thickness of the outer vertical flange of the T section;
- \( b_p \) is the width of the annular plate ring;
- \( b_f \) is the height (flange width) of the outer vertical flange of the T section;
- \( A \) is the cross-sectional area of the T-section ring beam;
- \( x_c \) is the distance between the centroid of the T-section and its inner edge;
- \( I_r \) is the second moment of area of the T-section about its radial axis;
- \( I_z \) is the second moment of area of the T-section about its vertical axis;
- \( I_t \) is the uniform torsion constant for the T-section;
- \( \gamma_{M1} \) is the partial factor, see 2.9.2.

8.4 Limit state verifications

8.4.1 Uniformly supported transition junctions

(1) Where the silo has been analysed using a computer analysis, the procedures of EN 1993-1-6 should be used. Where the computer analysis does not include a buckling analysis, section 8.3 may be used to provide the buckling resistances for the limit state verification in EN 1993-1-6.

(2) Where the silo is supported on a skirt extending to a uniform foundation (see 5.4.2) and the calculations of 8.2 have been carried out, the transition junction may be deemed to be subject only to a uniform circumferential membrane stress \( \sigma_{\theta,Ed} \) as determined in 8.2.2 (10). The following limit state verifications should then be carried out.

(3) Where the plastic limit state is assessed using an elastic evaluation, the plastic limit state for the junction should be verified using:

\[
\sigma_{\theta,Ed} \leq f_{p,Rd}
\]  

... (8.45)

where:

- \( \sigma_{\theta,Ed} \) is the design value of the stress taken from 8.2.2 (10);
- \( f_{p,Rd} \) is the design value of the plastic resistance taken from 8.3.2.2.

(4) Where the plastic limit state is assessed using a plastic evaluation, the plastic limit state for the junction should be verified using:

\[
n_{\phi,h,Ed} \leq n_{\phi,h,Rd}
\]  

... (8.46)

where:
\( n_{4h,Ed} \) is the design value of the meridional membrane stress resultant at the top of the hopper;

\( n_{4h,Rd} \) is the design value of the plastic resistance taken from 8.3.2.3.

(5) The in-plane buckling limit state for the junction should be verified using:

\[
\sigma_{\theta,Ed} \leq \sigma_{p,Rd}
\]

where:

\( \sigma_{\theta,Ed} \) is the design value of the stress taken from 8.2.2 (10);

\( \sigma_{p,Rd} \) is the design value of the in-plane buckling resistance taken from 8.3.3.

(6) The limit state verification against in-plane buckling may be omitted if both of the following conditions are met:

- the cone half angle \( \beta \) is greater than \( \beta_{\text{lim}} \) and there is a cylinder above the ring;
- where the cylinder has a height \( L \) less than \( L_{\text{min}} = k_L \sqrt{rt} \), the upper boundary of the cylinder is restrained against out-of-round displacements by a ring with a flexural rigidity \( EI_x \) about its vertical axis (circumferential bending) greater than:

\[
EI_{\text{c, min}} = k_R E (rt)^2 \sqrt{(rt)}
\]

where:

\( r \) is the thickness of the thinnest strake in the cylinder.

**NOTE 1:** The National Annex may choose the values of \( \beta_{\text{lim}} \), \( k_L \), and \( k_R \). The values \( \beta_{\text{lim}} = 10^\circ \), \( k_L = 10 \) and \( k_R = 0.04 \) are recommended.

**NOTE 2:** The requirement that the top of the cylinder should be restrained to remain circular is only relevant for short cylinders above the transition ring, since taller cylinders provide sufficient restraint against this mode of buckling without being themselves restrained to remain circular.

(7) The out-of-plane buckling limit state for the junction should be verified using:

\[
\sigma_{\theta,Ed} \leq \sigma_{op,Rd}
\]

where:

\( \sigma_{\theta,Ed} \) is the design value of the stress taken from 8.2.2 (10);

\( \sigma_{op,Rd} \) is the appropriate design value of the out-of-plane buckling resistance taken from 8.3.4.

8.4.2 Transition junction ring girder

(1) Where the silo has been analysed using a computer analysis, the procedures of EN 1993-1-6 should be used. Where the computer analysis does not include a buckling analysis, section 8.3 may be used to provide the buckling resistances for the limit state verification in EN 1993-1-6.

(2) Where the silo is discretely supported, so that the transition junction acts as a ring girder with circumferential membrane stresses which vary across the section and around the circumference, this variation should be taken into account in the limit state verifications. Where the calculations of 8.2 have been carried out, the following limit state verifications should be undertaken.

(3) The plastic limit state for the junction should use the evaluated stress \( \sigma_{\theta,Ed} \) from 8.2.3 (8) and should be verified using:
where:

\[\sigma_{n0,Ed} \leq f_{p,Rd}\]  

... (8.50)

\[\sigma_{n0,Ed}\] is the design value of the stress taken from 8.2.3 (8);
\[f_{p,Rd}\] is the design value of the plastic resistance taken from 8.3.2.2.

(4) The in-plane buckling limit state for the junction should use the evaluated stress \(\sigma_{\theta,Ed}\) from 8.2.3 (9) and should be verified using:

\[\sigma_{\theta,Ed} \leq \sigma_{ip,Rd}\]  

... (8.51)

where:

\[\sigma_{\theta,Ed}\] is the design value of the stress taken from 8.2.3 (9);
\[\sigma_{ip,Rd}\] is the design value of the in-plane buckling resistance taken from 8.3.3.

(5) The limit state verification against in-plane buckling may be omitted if both of the following conditions are met:

- the cone half angle \(\beta\) is greater than \(\beta_{lim}\) and there is a cylinder above the ring;
- where the cylinder has a height \(L\) less than \(L_{min} = k_L \sqrt{r}\), the upper boundary of the cylinder is restrained against out-of-round displacements by a ring with a flexural rigidity \(EI_k\) about its vertical axis (circumferential bending) greater than:

\[EI_{L_{min}} = k_R E (\pi r)^2 \sqrt{d/r}\]  

... (8.52)

where:

- \(t\) is the thickness of the thinnest strake in the cylinder;
- \(L\) is the height of the shell wall above the ring.

NOTE 1: The National Annex may choose the values of \(\beta_{lim}\), \(k_L\) and \(k_R\). The values \(\beta_{lim} = 10^\circ\), \(k_L = 10\) and \(k_R = 0.04\) are recommended.

NOTE 2: The requirement that the top of the cylinder should be restrained to remain circular is only relevant for short cylinders above the ring, since taller cylinders provide sufficient restraint against this mode of buckling without being themselves restrained to remain circular.

(6) The out-of-plane buckling limit state for the junction should use the evaluated stress \(\sigma_{\theta,Ed}\) from 8.2.3 (9) and should be verified using:

\[\sigma_{\theta,Ed} \leq \sigma_{op,Rd}\]  

... (8.53)

where:

\[\sigma_{\theta,Ed}\] is the design value of the stress taken from 8.2.3 (9);
\[\sigma_{op,Rd}\] is the design value of the out-of-plane buckling resistance taken from 8.3.4.

8.5 Considerations concerning support arrangements for the junction

8.5.1 Skirt supported junctions

(1) Where the silo is supported on a skirt extending to a uniform foundation (see 5.4.2), the transition junction may be deemed to carry only circumferential membrane stresses.
(2) The skirt should be checked for resistance to buckling under axial compression, including the effects of openings in the skirt.

8.5.2 Column supported junctions and ring girders

(1) Where the silo is supported on discrete supports or columns, and a transition ring girder is used to distribute column forces into the shell, the junction and ring girder should satisfy the conditions given in 8.2.3 and 8.4.2.

(2) Where a transition ring girder is formed by bolting together an upper and lower half, each attached to a different shell segment, the bolts should be proportioned to resist transmission of the full design value of the circumferential force to be carried in the upper ring segment, taking proper account of bending actions in the ring.

8.5.3 Base ring

(1) A silo that is continuously supported on the ground should be provided with a base ring and anchorage details.

(2) The circumferential spacing of anchorage bolts or other attachment points should not exceed $\sqrt{\frac{r}{t}}$, where $t$ is the local thickness of the shell plate.

(3) The base ring should have a flexural rigidity $EI_z$ about a vertical axis (to resist circumferential bending) greater than the minimum value $EI_{z,min}$ given by:

$$EI_{z,min} = k E r^3$$

where $t$ should be taken as the thickness of the wall strake adjacent to the base ring.

NOTE: The National Annex may choose the value of $k$. The value $k = 0.10$ is recommended.
9 Design of rectangular and planar-sided silos

9.1 Basis

(1) A rectangular silo should be designed either as a stiffened box in which the structural action is predominantly bending, or as a thin membrane structure in which the action is predominantly membrane stresses developing after large deformations.

(2) Where the box is designed for bending action, the joints should be designed to ensure that the connectivity assumed in the stress analysis is achieved in the execution.

9.2 Classification of structural forms

9.2.1 Unstiffened silos

(1) A structure formed from flat steel plates without attached stiffeners should be termed an 'unstiffened box'.

(2) A structure stiffened only along joints between plates which are not coplanar should also be termed an 'unstiffened box'.

9.2.2 Stiffened silos

(1) A structure formed from flat plates to which stiffeners are attached within the plate area should be termed a 'stiffened box'. The stiffeners may be circumferential or vertical or orthogonal (two directional).

![Figure 9.1: Plan view of tied rectangular box silo](image)

9.2.3 Silos with ties

(1) Silos with ties may be square or rectangular.

NOTE: Some typical structural components for a 3-panel square (single cell) silo are shown in figures 9.1 and 9.2.
9.3 Resistance of unstiffened vertical walls

(1) The resistance of vertical walls should be evaluated in accordance with EN 1993-1-7. Alternatively, the provisions set out in 9.4 may be deemed to satisfy the provisions of that Standard.

(2) The resistance of vertical walls should be evaluated considering both the membrane and plate bending actions.

(3) The actions on the unstiffened plate may be divided into the following categories:
   - bending as a 2D plate from the stored material;
   - stresses resulting from diaphragm action;
   - local bending action from the stored material and/or equipment.

9.4 Resistance of silo walls composed of stiffened and corrugated plates

9.4.1 General

(1) The resistance of unstiffened parts of vertical walls should be evaluated in accordance with the provisions set out in 9.4. The resistance evaluation should consider both membrane and plate bending actions.

(2) Horizontally corrugated plates should be designed for (see figure 9.3):
   - general bending action from pressures due to the stored material;
   - stresses resulting from their diaphragm action;
   - local bending action from the stored material and/or equipment.

(3) Effective bending properties and bending resistance of the stiffened plates should be derived in accordance with the provisions for trapezoidal sheeting with intermediate stiffeners in EN 1993-1-3.

(4) The design of the stiffeners should be made in accordance with member design given in EN 1993-1-1 and EN 1993-1-3, taking into account the compatibility of the stiffeners with the wall elements, the effect of the eccentricity of the sheeting in relation to the stiffener-axes, and the flexural continuities of wall elements and intersection horizontal and vertical stiffeners. Stresses normal to the longitudinal axis arising in stiffeners, which intersect structurally continuous wall-elements, should be taken into account additionally in the member design.

(5) The load transfer of vertical stiffeners to base boundary elements should be designed in accordance with the specific element and the given foundation resistance.
(6) Shear stiffness and resistance should be derived by testing or appropriate theoretical expressions.

(7) Unless a more precise method is available, the shear buckling strength may be assessed using 5.3.4.6 and treating the radius of the shell as infinite.

(8) Where testing is used, the relevant shear stiffness may be taken as the secant value achieved at 2/3 of the ultimate shear strength, see figure 9.4.

![Figure 9.3: Typical section on a vertical plane through the corrugated wall of rectangular silo](vertical section)

![Figure 9.4: Shear response of corrugated wall](diagram)

**Figure 9.3:** Typical section on a vertical plane through the corrugated wall of rectangular silo

**Figure 9.4:** Shear response of corrugated wall

### 9.4.2 General bending from direct action of the stored material

(1) Bending should be considered where horizontal bending can arise resulting from horizontal pressure or from horizontal pressure combined with wall friction.

(2) For bending from horizontal pressure alone, the calculation should be based on the effective properties as given by EN 1993-1-3.

(3) For bending arising from horizontal pressure combined with wall friction, the calculation may be based on the concept outlined in figure 9.5, where the wall section between Point A and Point B is considered as a cross-section in bending under the action of the combined pressure $p_g$. The stresses arising from the moment should be combined with those from the axial force arising from the stored material pressure on the adjacent perpendicular walls (see 9.4.3).
**NOTE:** This calculation is conventional and well established. However, it may be noted that the continuity of strain between adjacent wall sections is neglected.

![Diagram](image)

**Figure 9.5:** Bending resulting from combined horizontal pressure and friction (vertical section)

![Diagram](image)

**Figure 9.6:** Membrane forces induced in walls by solids pressures or wind loading

### 9.4.3 Membrane stresses from diaphragm action

1. The stresses result from pressure of stored material and/or wind on the perpendicular neighbouring walls, see figure 9.6.

2. As a simple rule, pressures from the stored material may be taken as only normal pressures (neglecting wall friction).

3. Direct and shear stresses from wind action may be determined using either hand calculations or a finite element calculation.
9.4.4 Local bending action from the stored material and/or equipment

(1) The possibility of deleterious local bending effects in any structural element due to the local stored material pressure should be taken into account.

**NOTE:** In the situation shown in figure 9.7, the structural check on the plate element CD may be critical.

**Figure 9.7: Possible local bending actions**

9.5 Silos with internal ties

9.5.1 Forces in internal ties due to solids pressure on them

(1) The force exerted by the stored bulk solid on the tie should be evaluated.

(2) Unless more precise calculations are made, the force exerted by the solid \( q_t \) per unit length of tie may be approximated by:

\[
q_t = C_i p_v b
\]  \hspace{1cm} ... (9.1)

with:

\[
C_i = \frac{C_s \beta}{k_L}
\]  \hspace{1cm} ... (9.2)

where:

- \( p_v \) is the vertical pressure within the stored material at the tie level;
- \( b \) is the maximum horizontal width of the tie;
- \( C_i \) is the load magnification factor;
- \( C_s \) is the shape factor for the tie cross-section;
- \( k_L \) is the loading state factor;
- \( \beta \) is the tie location factor, that depends on the position of the tie within the silo cell (see figures 9.8 and 9.9).
(3) The shape factor $C_s$ should be taken as follows:
- for circular smooth sections: $C_s = C_{sc}$
- for round rough or square sections: $C_s = C_{sn}$

**NOTE:** The National Annex may choose the values of $C_{sc}$ and $C_{sn}$. The values $C_{sc} = 1.0$ and $C_{sn} = 1.2$ are recommended.

(4) The loading state factor $k_L$ should be taken as follows:
- for bulk solids filling: $k_L = k_{LF}$
- for bulk solids discharge: $k_L = k_{LE}$

**NOTE:** The National Annex may choose the value of $k_L$. The value $k_{LF} = 4.0$ and $k_{LE} = 2.0$ are recommended.

![Figure 9.8: Evaluation of factor $\beta$ for internal ties](image)

**9.5.2 Modelling of ties**

(1) Ties should be classified according to the principle means by which they support the loads. A tie should be classed as a cable if it has negligible bending stiffness. It should be classed as a rod if it has both axial stiffness and significant bending stiffness. The analysis of the tie should be appropriate to the structural response of the tie section.

(2) Where the tie is a rod, account should be taken of bending moments in addition to the axial tension.

(3) A geometrically non-linear calculation procedure should be used to determine the force $N$ (and for rods, the moments $M$) in the tie. This analysis should take account of the real boundary conditions and the stiffness of the silo wall (Figure 9.10).

(4) The design values of axial tension $N$ and moment $M$ should be taken as the values in the tie at the connection to the wall.

(5) The initial sag of the tie should be agreed between the client, the designer and the fabricator. For cables (negligible flexural stiffness), the initial sag should not be greater than $k_s L$ where $L$ is the length of the tie.

**NOTE 1:** The National Annex may choose the value of $k_s$. The value $k_s = 0.01$ is recommended.
NOTE 2: In past practice, the sag has often been set at 0.02L. The smaller value that is recommended here is needed to obtain a relationship between pressures and induced forces that is close linear in the operating range.

(6) The attachment details for ties should take account of both the vertical and the horizontal components of the tie tension at the point of attachment.

![Figure 9.9: Corner ties for which $\beta = 0.7$](image)

![Figure 9.10: Force development in a tie](image)

### 9.5.3 Load cases for tie attachments

(1) The analysis of the tie should take account of:
- actions from the stored material;
- forces transmitted to the ties due to deformations of the walls from other load cases.

(2) Two load cases for the attachment forces and moments from a tie should be checked as follows:
   a) load case 1: the values of $q_t$ and $N$ evaluated in 9.5.1 and 9.5.2.
   b) load case 2: an increased value of transverse load $1.2q_t$ and a reduced value of tie tension $0.7N$, where $q_t$ and $N$ have been evaluated according to 9.5.1 and 9.5.2.

### 9.6 Strength of pyramidal hoppers

(1) Pyramidal hoppers (Figure 9.12) should be treated as box structures, using the provisions of EN 1993-1-7. These may be deemed to be met by the provisions of 9.3 and 9.4 for walls, together with the following approximate methods.

(2) The bending moments and membrane forces may be determined using numerical methods, in accordance with EN 1993-1-6 and EN 1993-1-7. The bending moments in the trapezoidal plates of the hopper may alternatively be evaluated using the following approximate relationships.

(3) An equilateral triangle $ABE$ of area $A$ should be drawn on the hopper plate $ABCD$, and the radius of the equivalent equal-area circle should be determined using:
where:

\[ a \] is the horizontal length of the upper edge of the plate, see figure 9.11.

\[ r_{eq} = \frac{A}{\sqrt{\pi}} = 0.37 \ a \] \hspace{1cm} \text{... (9.3)}

**Figure 9.11: Simple model for bending of trapezoidal plates**

(4) The reference bending moment \( M_0 \) should be determined using:

\[ M_0 = \frac{3}{16} p_n r_{eq}^2 = 0.026 \ p_n a^2 \] \hspace{1cm} \text{... (9.4)}

where:

\[ p_n \] is the mean normal pressure on the trapezoidal plate.

(5) Where the trapezoidal plate has edges that may be treated as simply supported, the design value of bending moment may be taken as:

\[ M_{s,Ed} = M_0 \] \hspace{1cm} \text{... (9.5)}

(6) Where the trapezoidal plate has edges that may be treated as clamped, the bending moment at the plate centre \( M_{c,Ed} \) and the bending moment at the edge \( M_{e,Ed} \) may be taken as:

\[ M_{s,Ed} = 0.80 \ M_0 \] \hspace{1cm} \text{... (9.6)}

\[ M_{e,Ed} = 0.53 \ M_0 \] \hspace{1cm} \text{... (9.7)}
9.7 Vertical stiffeners on box walls

(1) Vertical stiffeners on the walls of a box should be designed for:
   - the permanent actions;
   - the normal pressures on the wall due to bulk solids;
   - the friction forces on the wall;
   - the variable actions from the roof;
   - the axial forces arising from contributions from the diaphragm action in the walls.

(2) The eccentricity of the friction forces from the plate and stiffener centrelines may be neglected.

9.8 Serviceability limit states

9.8.1 Basis

(1) The serviceability limit states for rectangular silo walls should be taken as follows:
   - deformations or deflections that adversely affect the effective use of the structure;
   - deformations, deflections, vibration or oscillation that causes damage to both structural and non-structural elements.

(2) Deformations, deflections and vibrations should be limited to meet the above criteria.

(3) Specific limiting values, appropriate to the intended use, should be agreed between the designer, the client and the relevant authority, taking account of the intended use and the nature of the solids to be stored.

9.8.2 Deflections

(1) The limiting value for global lateral deflection should be taken as the lesser of:

$$\delta_{\text{max}} = k_1 H$$  \hspace{1cm} ... (9.8)

$$\delta_{\text{max}} = k_2 t$$  \hspace{1cm} ... (9.9)

where:
$H$ is the height of the structure measured from the foundation to the roof;
$t$ is the thickness of the thinnest plate in the wall.

**NOTE:** The National Annex may choose the values of $k_1$ and $k_2$. The values $k_1 = 0.02$ and $k_2 = 10$ are recommended.

(2) The maximum deflection $\delta_{\text{max}}$ within any panel section relative to its edges should be limited to:

$$\delta_{\text{max}} < k_3 L$$

... (9.10)

where $L$ is the shorter dimension of the rectangular plate.

**NOTE:** The National Annex may choose the value of $k_3$. The value $k_3 = 0.05$ is recommended.
Annex A: [Informative]
Simplified rules for circular silos in Consequence Class 1

For circular silos with cylindrical walls in Consequence Class 1, this simplified treatment permits a
design based only on the ultimate limit state and with a restricted number of load cases being
addressed.

A.1 Action combinations for Consequence Class 1

The following simplified action combinations may be considered for silos in Consequence Class 1:
- Filling
- Discharge
- Wind when empty
- Filling, combined with wind

A simplified treatment of wind loading is permitted.

A.2 Action effect assessment

(1) When designing to the expressions given in this annex, the membrane stresses should be
increased by the factor $k_M$ to account for local bending effects.

NOTE: The National Annex may choose the value of $k_M$. The value $k_M = 1,1$ is recommended.

(2) When designing to the expressions given in this annex, the hopper and ring forces should be
increased by the factor $k_h$ to account for unsymmetrical and ring bending effects.

NOTE: The National Annex may choose the value of $k_h$. The value $k_h = 1,2$ is recommended.

A.3 Ultimate limit state assessment

A.3.1 General

(1) The limited provisions given here permit a faster assessment of a design, but they are often
more conservative than the more complete provisions of the standard.

A.3.2 Isotropic welded or bolted cylindrical walls

A.3.2.1 Plastic limit state

(1) Under internal pressure and all relevant design loads, the design resistance should be
determined at every point using the variation in internal pressure, as appropriate, and the local
strength to resist it.

(2) At every point in the structure the design membrane stress resultants $n_{x,Ed}$ and $n_{θ,Ed}$ (both
taken as tension positive) should satisfy the condition:

$$\sqrt{n_{x,Ed}^2 - n_{x,Ed}n_{θ,Ed} + n_{θ,Ed}^2} \leq \frac{f_{y}}{f_{M}}$$

... (A.1)

where:

$n_{x,Ed}$ is the vertical membrane stress resultant (force per unit width of shell wall) derived by analysis from the design values of the actions (loads);
$n_{\theta,Ed}$ is the circumferential membrane stress resultant (force per unit width of shell wall) derived by analysis from the design values of the actions (loads);

$f_y$ is the yield strength of the shell wall plate;

$\gamma_{M0}$ is the partial factor against the plastic limit state.

(3) At every bolted joint in the structure the design stress resultants should satisfy the conditions against net section failure:

- for meridional resistance $n_{x,Ed} \leq f_u \cdot t / \gamma_{M2}$ ... (A.2)

- for circumferential resistance $n_{\theta,Ed} \leq f_u \cdot t / \gamma_{M2}$ ... (A.3)

where:

$f_u$ is the ultimate strength of the shell wall plate;

$\gamma_{M2}$ is the partial factor against rupture (=1.25).

(4) The design of connections should be carried out in accordance with EN 1993-1-8 or EN 1993-1-3. The effect of fastener holes should be taken into account according to EN 1993-1-1 using the appropriate requirements for tension or compression or shear as appropriate.

(5) The design resistance at lap joints in welded construction $f_{e,Rd}$ is given by the fictitious strength criterion:

$$f_{e,Rd} = j f_y / \gamma_{M0}$$ ... (A.4)

where $j$ is the joint efficiency factor.

(6) The joint efficiency of lap joint welded details with full continuous fillet welds should be taken as $j = j_1$.

NOTE: The National Annex may choose the value of $j_i$. The recommended values of $j_i$ are given in the table below for different joint configurations. The single welded lap joint should not be used if more than 20% of $\sigma_{\text{cr}}$ in expression 5.4 derives from bending moments.

<table>
<thead>
<tr>
<th>Joint type</th>
<th>Sketch</th>
<th>Value of $j_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double welded lap</td>
<td><img src="DoubleWeldedLap.png" alt="Sketch" /></td>
<td>$j_1 = 1.0$</td>
</tr>
<tr>
<td>Single welded lap</td>
<td><img src="SingleWeldedLap.png" alt="Sketch" /></td>
<td>$j_2 = 0.35$</td>
</tr>
</tbody>
</table>

A.3.2.2 Axial compression

(1) Under axial compression, the design resistance should be determined at every point in the shell. The design should ignore the vertical variation of the axial compression, except where the provisions of EN 1993-1-6 make provision for this. In buckling calculations, compressive membrane forces should be treated as positive to avoid widespread use of negative numbers.

(2) Where a horizontal lap joint is used, causing eccentricity of the axial force in passing through the joint, the value of $\alpha$ given below should be reduced to 70% of its previous value if the eccentricity of the middle surface of the plates to one another exceeds $t$ and the change in plate
thickness at the joint is not more than \( t_{14} \), where \( t \) is the thickness of the thinner plate at the joint. Where the eccentricity is smaller than this value, or the change in plate thickness is greater, no reduction need be made in the value of \( \alpha \).

(3) The elastic imperfection reduction factor \( \alpha \) should be found as:

\[
\alpha = \frac{0.62}{1 + 0.035 \left( \frac{r}{t} \right)^{0.72}} \quad \text{... (A.5)}
\]

where:
- \( r \) is the radius of the silo wall;
- \( t \) is the thickness of the wall plate at the location being calculated.

(4) The critical buckling stress \( \sigma_{x,Rcr} \) at any point in the isotropic wall should be calculated as:

\[
\sigma_{x,Rcr} = 0.605 E \frac{t}{r} \quad \text{... (A.6)}
\]

(5) The characteristic buckling stress should be found as:

\[
\sigma_{x,Rk} = \chi_x f_y \quad \text{... (A.7)}
\]

in which:

\[
\chi_x = 1 \quad \text{when} \quad \bar{\lambda}_x \leq \bar{\lambda}_0 \quad \text{... (A.8)}
\]

\[
\chi_x = 1 - 0.6 \left( \frac{\bar{\lambda}_x - \bar{\lambda}_0}{\bar{\rho} - \bar{\lambda}_0} \right) \quad \text{when} \quad \bar{\lambda}_0 < \bar{\lambda}_x < \bar{\rho} \quad \text{... (A.9)}
\]

\[
\chi_x = \frac{\alpha}{\bar{\rho}^2} \quad \text{when} \quad \bar{\rho} \leq \bar{\lambda}_x \quad \text{... (A.10)}
\]

with:

\[
\bar{\lambda}_x = \sqrt{\frac{K_x}{\sigma_{x,Rk}}} \quad \bar{\lambda}_0 = 0.2 \quad \text{and} \quad \bar{\rho} = \sqrt{2.5\alpha}
\]

(6) At every point in the structure the design membrane stress resultant \( n_{x,Ed} \) (compression positive) should satisfy the condition:

\[
n_{x,Ed} \leq t \sigma_{x,Rk} / \chi_M \quad \text{... (A.11)}
\]

where \( \chi_M \) is given by 2.9.2.

**NOTE:** The National Annex may choose the value of \( \chi_M \). The value \( \chi_M = 1.1 \) is recommended.
(7) The maximum permitted measurable imperfection, using the procedures of EN 1993-1-6 and excluding measurements across lap joints, should be found as:

\[ \Delta w_{ed} = 0.0375 \sqrt{rt} \]  

... (A.12)

(8) The design of the shell against buckling under axial compression above a local support, near a bracket (e.g. to support a conveyor gantry), and near an opening should be undertaken as stipulated in 5.6.

A.3.2.3 External pressure, internal partial vacuum and wind

(1) For uniform partial internal vacuum (external pressure), where there is a structurally connected roof, the critical buckling external pressure \( p_{n,RCU} \) for the isotropic wall should be found as:

\[ p_{n,RCU} = 0.92E \left( \frac{r}{l} \right)^{2.5} \]  

... (A.13)

where:
- \( r \) is the radius of the silo wall;
- \( t \) is the thickness of the thinnest part of the wall;
- \( \ell \) is the height between stiffening rings or boundaries.

(2) The design value of the maximum external pressure \( p_{n,Ed} \) acting on the structure under the combined actions of wind and partial vacuum should satisfy the condition:

\[ p_{n,Ed} \leq \alpha_n \frac{p_{n,RCU}}{\gamma_{M1}} \]  

... (A.14)

**NOTE:** The National Annex may choose the values of \( \alpha_n \) and \( \gamma_{M1} \). The values \( \alpha_n = 0.5 \) and \( \gamma_{M1} = 1.1 \) are recommended.

(3) If the upper edge of the cylinder is not connected to the roof, this simple procedure should be replaced with that of 5.3.

A.3.3 Conical welded hoppers

(1) A simple design procedure may be used provided that both the following conditions are met:

a) An enhanced partial factor is used for the hopper of \( \gamma_{M0} = \gamma_{M0g} \);

b) No local meridional stiffeners or supports are attached to the hopper wall near the transition junction.

**NOTE:** The National Annex may choose the value of \( \gamma_{M0g} \). The value \( \gamma_{M0g} = 1.4 \) is recommended.

(2) Where the only loading under consideration is gravity and flow loading from the stored solid, the meridional force per unit circumference \( n_{qh,Ed} \) caused by the symmetrical pressures defined in EN 1991-4 that must be transmitted through the transition joint should be evaluated using global equilibrium, see Figure A.1. The design value of the local meridional force per unit circumference \( n_{qh,Ed} \) allowing for the possible non-uniformity of the loading, should then be obtained as

\[ n_{qh,Ed} = g_{asym} n_{qh,Ed,s} \]  

... (A.15)
where:

\[ n_{h,Ed,s} \] is the design value of the meridional membrane force per unit circumference at the top of the hopper obtained assuming the hopper loads are entirely symmetrical;

\[ g_{\text{asym}} \] is the unsymmetrical stress augmentation factor.

**NOTE:** Expressions for \( n_{h,Ed,s} \) may be found in Annex B. The National Annex may choose the value of \( g_{\text{asym}} \). The value \( g_{\text{asym}} = 1.2 \) is recommended.

---

**Figure A.1: Hopper global equilibrium**

---

(3) The design value of the meridional membrane tension at the hopper top \( n_{h,Ed} \) should satisfy the condition:

\[
n_{h,Ed} \leq k_r t_f / \gamma_{M2} \tag{A.16}
\]

where:

- \( t \) is the thickness of the hopper;
- \( f_f \) is the tensile strength;
- \( \gamma_{M2} \) is the partial factor for rupture.

**NOTE:** The National Annex may choose the value of \( k_r \). The value \( k_r = 0.90 \) is recommended. The National Annex may also choose the value of \( \gamma_{M2} \). The value \( \gamma_{M2} = 1.25 \) is recommended.

---

**A.3.4 Transition junction**

(1) This simplified design method may be used on silos of Consequence Class 1 where the junction consists of a cylindrical and conical section, with or without an annular plate or similarly compact ring at the junction, see figure A.2.
(2) The total effective area of the ring $A_{et}$ should be found from:

$$A_{et} = A_p + 0.4 \sqrt{r} \left( t_c^{3/2} + t_s^{3/2} + \frac{t_h^{3/2}}{\cos \beta} \right)$$  \hspace{1cm} (A.17)

where:

- $r$ is the radius of the silo cylinder wall;
- $t_c$ is the thickness of the cylinder;
- $t_s$ is the thickness of the skirt;
- $t_h$ is the thickness of the hopper;
- $\beta$ is the cone apex half angle of the hopper;
- $A_p$ is the area of the ring at the junction.

(3) The design value of the circumferential compressive force $N_{th,Ed}$ developed in the junction should be determined from:

$$N_{th,Ed} = n_{th,Ed} \ r \sin \beta$$  \hspace{1cm} (A.18)

where:

- $n_{th,Ed}$ is the design value of the meridional tension per unit circumference at the top of the hopper, see Figure A.1 and expression A.15.
(4) The mean circumferential stress in the ring should satisfy the condition:

$$\frac{N_{b,j,k}}{A_o} \leq \frac{f_y}{\gamma_{M0}}$$

where:

- $f_y$ is the lowest yield strength of the ring and shell materials;
- $\gamma_{M0}$ is the partial factor for plasticity.

**NOTE:** The National Annex may choose the value of $\gamma_{M0}$. The value $\gamma_{M0} = 1.0$ is recommended.
Annex B: [Informative]
Expressions for membrane stresses in conical hoppers

The expressions given here permit membrane theory stress analyses to be undertaken for cases which
are not obtainable in standard texts on shells or silo structures. Membrane theory expressions
accurately predict the membrane stresses in the body of the hopper (i.e. at points not adjacent to the
transition or support) provided that the applied loadings are according to patterns defined in EN 1991-4.

Coordinate system with origin for \( z \) at the apex

Vertical height of hopper \( h \) and cone apex half angle \( \beta \)

**B.1 Uniform pressure \( p_0 \) with wall friction \( \mu p_0 \)**

\[
\sigma_\theta = \frac{p_0 z}{t} \left( \tan \beta \right) \left( \cos \beta \right) \quad \ldots \quad (B.1)
\]

\[
\sigma_\phi = \frac{p_0 z}{2t} \left( \tan \beta + \mu \right) \left( \cos \beta \right) \quad \ldots \quad (B.2)
\]

**B.2 Linearly varying pressure from \( p_1 \) at apex to \( p_2 \) at transition with wall friction \( \mu p \)**

\[
p = p_1 + \frac{z}{h} (p_2 - p_1) \quad \ldots \quad (B.3)
\]

\[
\sigma_\theta = \left\{ p_1 + \frac{z}{h} (p_2 - p_1) \right\} \frac{z}{t} \left( \tan \beta \right) \left( \cos \beta \right) \quad \ldots \quad (B.4)
\]

\[
\sigma_\phi = \left\{ 3p_1 + \frac{2z}{h} (p_2 - p_1) \right\} \frac{z}{6t} \left( \tan \beta + \mu \right) \left( \cos \beta \right) \quad \ldots \quad (B.5)
\]

For \( \mu = 0 \), the maximum von Mises equivalent stress occurs in the body of the cone if \( p_2 < 0.48 \ p_1 \)
at the height:

\[
z = 0.52 \left( \frac{p_1}{p_2 - p_1} \right) h \quad \ldots \quad (B.6)
\]

**B.3 “Radial stress field” with triangular switch stress at the transition**

\[
p = p_1 \frac{z}{h_1} \quad \text{for } 0 < z < h_1 \quad \ldots \quad (B.7)
\]
\[ p = \frac{p_1 (h-z) - p_2 (h - z)}{h - h_1} \] for \( h_1 < z < h \) ... (B.8)

\[ \sigma_\theta = \frac{p_1 \left( \frac{z^2}{3h} \right) \left( \tan \beta \right)}{\cos \beta} \] for \( 0 < z < h_1 \) ... (B.9)

\[ \sigma_\theta = \left[ \frac{z p_1 (h-z) - p_2 (h - z)}{h (h - h_1)} \right] \left( \tan \beta \right) \] for \( h_1 < z < h \) ... (B.10)

\[ \sigma_\theta = \frac{p_2 z^2}{3h_1} \left( \frac{\tan \beta + \mu}{\cos \beta} \right) \] for \( 0 < z < h_1 \) ... (B.11)

\[ \sigma_\theta = \left[ \frac{2 \left( p_2 - p_1 \right) + (3z^2 - h_1^2) (hp_1 - h_1 p_2)}{6zh(h - h_1)} \right] \left( \frac{\tan \beta + \mu}{\cos \beta} \right) \] for \( h_1 < z < h \) ... (B.12)

in which \( p_1 \) is the pressure at a height \( h_1 \) above the apex and \( p_2 \) is the pressure at the transition.

### B.4 General hopper theory pressures

The pressure pattern may be defined in terms of the normal pressure \( p \) with accompanying wall frictional traction \( \mu p \) as:

\[ p = Fq \] ... (B.13)

\[ q = \frac{\gamma h}{n-1} \left[ \left( \frac{z}{h} \right) - \left( \frac{z}{h} \right)^n \right] + q_t \left( \frac{z}{h} \right)^n \] ... (B.14)

with:

\[ n = 2(F \mu \cot \beta + F - 1) \] ... (B.15)

\( F \) is the ratio of wall pressure \( p \) to vertical stress in the solid \( q \) and \( q_t \) is the mean vertical stress in the solid at the transition:

\[ \sigma_\theta = \left[ \frac{\gamma h}{(n-1)} \left( \frac{z}{h} \right)^2 \right] + \left( \frac{q_t - \gamma h}{(n-1)} \right) \left( \frac{F}{r} \right) \left( \frac{Fh}{t} \right) \left( \frac{\tan \beta}{\cos \beta} \right) \] ... (B.16)

\[ \sigma_\theta = \left[ \frac{\gamma h}{3(n-1)} \left( \frac{z}{h} \right)^2 + \frac{1}{(n+2)} \left( \frac{q_t - \gamma h}{(n-1)} \right) \left( \frac{F}{r} \right) \left( \frac{Fh}{t} \right) \left( \frac{\tan \beta + \mu}{\cos \beta} \right) \right] \] ... (B.17)
Annex C: [Informative]
Distribution of wind pressure around circular silo structures

The distribution of wind pressures around a squat circular silo or ground-supported tank (see figure C.1) can be important to the assessment of anchorage requirements and wind buckling resistance. Values given in EN 1991-1-4 may not provide sufficient detail in certain cases.

The pressure variation around an isolated silo may be defined in terms of the circumferential coordinate $\theta$ with its origin at the windward generator (see figure C.2).

The circumferential variation of the pressure distribution (positive inward) on an isolated closed roof silo (see figure C.2) is given by:

$$C_p = -0.54 + 0.16(d_c/H) + \{0.28 + 0.04(d_c/H)\} \cos \theta + \{1.04 - 0.20(d_c/H)\} \cos 2\theta$$
$$+ \{0.36 - 0.05(d_c/H)\} \cos 3\theta - \{0.14 - 0.20(d_c/H)\} \cos 4\theta$$

... (C.1)

where $d_c$ is the diameter of the silo and $H$ its overall height ($H/d_c$ is the aspect ratio for the complete structure and its supports) (see figure C.1). For silos with $H/d_c$ less than 0.50, the values for $H/d_c = 0.50$ should be adopted. The pressure distribution should not be based on the cylinder height $H_c$.

The circumferential variation of the pressure distribution (positive inward) on a closed roof silo in a group (see figure C.3) may be taken as:

$$C_p = +0.20 + 0.60 \cos \theta + 0.27 \cos 2\theta - 0.05 \cos 3\theta - 0.13 \cos 4\theta + 0.13 \cos 6\theta$$
$$- 0.09 \cos 8\theta + 0.07 \cos 10\theta$$

... (C.2)

Figure C1: Wind loaded silo

Figure C2: Wind pressure variation around half circumference in isolated silo
Figure C3: Wind pressure variation around half circumference of silo in group

Where the silo does not have a closed roof, the following additional uniform values of internal underpressure coefficients $\Delta C_p$ should be added to the above, thus increasing the net stagnation inward pressure:

a) additional inward pressure on open top silo: $\Delta C_p = +0.6$;
b) additional inward pressure on a vented silo with a small opening: $\Delta C_p = +0.4$.

Note: $\Delta C_p$ is taken as positive inwards. For this case, the resultant of the external and internal pressure on the silo wall is close to zero on the leeward side of the silo.