EN 1993-1-6 (2007) (English): Eurocode 3: Design of steel structures - Part 1-6: Strength and stability of shell structures [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC]

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Eurocode 3 - Design of steel structures - Part 1-6: Strength and Stability of Shell Structures

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Eurocode 3 - Bemessung und Konstruktion von Stahlbauten - Teil 1-6: Festigkeit und Stabilität von Schalen

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Contents

Foreword

This European Standard EN 1993-1-6, Eurocode 3: Design of steel structures: Part 1-6 Strength and stability of shell structures, has been prepared by Technical Committee CEN/TC250 « Structural Eurocodes », the Secretariat of which is held by BSI. CEN/TC250 is responsible for all Structural Eurocodes.

This European Standard shall be given the status of a National Standard, either by publication of an identical text or by endorsement, at the latest by August 2007, and conflicting National Standards shall be withdrawn at latest by March 2010.

This Eurocode supersedes ENV 1993-1-6.

According to the CEN-CENELEC Internal Regulations, the National Standard Organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland and United Kingdom.

National annex for EN 1993-1-6

This standard gives alternative procedures, values and recommendations with notes indicating where national choices may have to be made. Therefore the National Standard implementing EN 1993-1-6 should have a National Annex containing all Nationally Determined Parameters to be used for the design of steel structures to be constructed in the relevant country.

National choice is allowed in EN 1993-1-6 through:

- 3.1.(4)
- 4.1.4 (3)
- 5.2.4 (1)
- 6.3 (5)
- 7.3.1 (1)
- 7.3.2 (1)
- 8.4.2 (3)
- 8.4.3 (2)
- 8.4.3 (4)
- 8.4.4 (4)
- 8.4.5 (1)
- 8.5.2 (2)
- 8.5.2 (4)
- 8.7.2 (7)
- 8.7.2 (16)
- 8.7.2 (18) (2 times)
- 9.2.1 (2)P

1. General

1.1 Scope

(1) EN 1993-1-6 gives basic design rules for plated steel structures that have the form of a shell of revolution.

(2) This Standard is intended for use in conjunction with EN 1993-1-1, EN 1993-1-3, EN 1993-1-4, EN 1993-1-9 and the relevant application parts of EN 1993, which include:

Part 3.1 for towers and masts;
Part 3.2 for chimneys;
Part 4.1 for silos;
Part 4.2 for tanks;
Part 4.3 for pipelines.

(3) This Standard defines the characteristic and design values of the resistance of the structure.

(4) This Standard is concerned with the requirements for design against the ultimate limit states of:

plastic limit;
cyclic plasticity;
buckling;
fatigue.

(5) Overall equilibrium of the structure (sliding, uplifting, overturning) is not included in this Standard, but is treated in EN 1993-1-1. Special considerations for specific applications are included in the relevant application parts of EN 1993.

(6) The provisions in this Standard apply to axisymmetric shells and associated circular or annular plates and to beam section rings and stringer stiffeners where they form part of the complete structure. General procedures for computer calculations of all shell forms are covered. Detailed expressions for the hand calculation of unstiffened cylinders and cones are given in the Annexes.

(7) Cylindrical and conical panels are not explicitly covered by this Standard. However, the provisions can be applicable if the appropriate boundary conditions are duly taken into account.

(8) This Standard is intended for application to steel shell structures. Where no standard exists for shell structures made of other metals, the provisions of this standards may be applied provided that the appropriate material properties are duly taken into account.

(9) The provisions of this Standard are intended to be applied within the temperature range defined in the relevant EN 1993 application parts. The maximum temperature is restricted so that the influence of creep can be neglected if high temperature creep effects are not covered by the relevant application part.

(10) The provisions in this Standard apply to structures that satisfy the brittle fracture provisions given in EN 1993-1-10.

(11) The provisions of this Standard apply to structural design under actions that can be treated as quasi-static in nature.

(12) In this Standard, it is assumed that both wind loading and bulk solids flow can, in general, be treated as quasi-static actions.

(13) Dynamic effects should be taken into account according to the relevant application part of EN 1993, including the consequences for fatigue. However, the stress resultants arising from dynamic behaviour are treated in this part as quasi-static.

(14) The provisions in this Standard apply to structures that are constructed in accordance with EN 1090-2.

(15) This Standard does not cover the aspects of leakage.

(16) This Standard is intended for application to structures within the following limits:

design metal temperatures within the range −50°C to +300°C;
radius to thickness ratios within the range 20 to 5000.

NOTE: It should be noted that the stress design rules of this standard may be rather conservative if applied to some geometries and loading conditions for relatively thick-walled shells.

1.2 Normative references

(1) This European Standard incorporates, by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text and the publications are listed hereafter. For dated references, subsequent amendments to or revisions of any

of these publications apply to this European Standard only when incorporated in it by amendment or revision. For undated references the latest edition of the publication referred to applies.

EN 1090-2	*Execution of steel structures and aluminium structures – Part 2: Technical requirements for steel structures;*
EN 1990	*Basis of structural design;*
EN 1991	*Eurocode 1: Actions on structures ;*
EN 1993	*Eurocode 3: Design of steel structures:*
Part 1.1:	*General rules and rules for buildings;*
Part 1.3:	*Cold formed thin gauged members and sheeting;*
Part 1.4:	*Stainless steels;*
Part 1.5:	*Plated structural elements;*
Part 1.9:	*Fatigue strength of steel structures;*
Part 1.10:	*Selection of steel for fracture toughness and through-thickness properties;*
Part 1.12:	*Additional rules for the extension of EN 1993 up to steel grades S 700*
Part 2:	*Steel bridges;*
Part 3.1:	*Towers and masts;*
Part 3.2:	*Chimneys;*
Part 4.1:	*Silos;*
Part 4.2:	*Tanks;*
Part 4.3:	*Pipelines;*
Part 5:	*Piling.*

1.3 Terms and definitions

The terms that are defined in EN 1990 for common use in the Structural Eurocodes apply to this Standard. Unless otherwise stated, the definitions given in ISO 8930 also apply in this Standard. Supplementary to EN 1993-1-1, for the purposes of this Standard, the following definitions apply:

1.3.1 Structural forms and geometry

1.3.1.1 shell

A structure or a structural component formed from a curved thin plate.

1.3.1.2 shell of revolution

A shell whose geometric form is defined by a middle surface that is formed by rotating a meridional generator line around a single axis through 2π radians. The shell can be of any length.

1.3.1.3 complete axisymmetric shell

A shell composed of a number of parts, each of which is a shell of revolution.

1.3.1.4 shell segment

A shell of revolution in the form of a defined shell geometry with a constant wall thickness: a cylinder, conical frustum, spherical frustum, annular plate, toroidal knuckle or other form.

1.3.1.5 shell panel

An incomplete shell of revolution: the shell form is defined by a rotation of the generator about the axis through less than 2π radians.

1.3.1.6 middle surface

The surface that lies midway between the inside and outside surfaces of the shell at every point. Where the shell is stiffened on either one or both surfaces, the reference middle surface is still taken as the middle surface of the curved shell plate. The middle surface is the reference surface for analysis, and can be discontinuous at changes of thickness or at shell junctions, leading to eccentricities that may be important to the shell structural behaviour.

1.3.1.7 junction

The line at which two or more shell segments meet: it can include a stiffener. The circumferential line of attachment of a ring stiffener to the shell may be treated as a junction.

1.3.1.8 stringer stiffener

A local stiffening member that follows the meridian of the shell, representing a generator of the shell of revolution. It is provided to increase the stability, or to assist with the introduction of local loads. It is not intended to provide a primary resistance to bending effects caused by transverse loads.

1.3.1.9 rib

A local member that provides a primary load carrying path for bending down the meridian of the shell, representing a generator of the shell of revolution. It is used to transfer or distribute transverse loads by bending.

1.3.1.10 ring stiffener

A local stiffening member that passes around the circumference of the shell of revolution at a given point on the meridian. It is normally assumed to have no stiffness for deformations out of its own plane (meridional displacements of the shell) but is stiff for deformations in the plane of the ring. It is provided to increase the stability or to introduce local loads acting in the plane of the ring.

1.3.1.11 base ring

A structural member that passes around the circumference of the shell of revolution at the base and provides a means of attachment of the shell to a foundation or other structural member. It is needed to ensure that the assumed boundary conditions are achieved in practice.

1.3.1.12 ring beam or ring girder

A circumferential stiffener that has bending stiffness and strength both in the plane of the shell circular section and normal to that plane. It is a primary load carrying structural member, provided for the distribution of local loads into the shell.

1.3.2 Limit states

1.3.2.1 plastic limit

The ultimate limit state where the structure develops zones of yielding in a pattern such that its ability to resist increased loading is deemed to be exhausted. It is closely related to a small deflection theory plastic limit load or plastic collapse mechanism.

1.3.2.2 tensile rupture

The ultimate limit state where the shell plate experiences gross section failure due to tension.

1.3.2.3 cyclic plasticity

The ultimate limit state where repeated yielding is caused by cycles of loading and unloading, leading to a low cycle fatigue failure where the energy absorption capacity of the material is exhausted.

1.3.2.4 buckling

The ultimate limit state where the structure suddenly loses its stability under membrane compression and/or shear. It leads either to large displacements or to the structure being unable to carry the applied loads.

1.3.2.5 fatigue

The ultimate limit state where many cycles of loading cause cracks to develop in the shell plate that by further load cycles may lead to rupture.

1.3.3 Actions

1.3.3.1 axial load

Externally applied loading acting in the axial direction.

1.3.3.2 radial load

Externally applied loading acting normal to the surface of a cylindrical shell.

1.3.3.3 internal pressure

Component of the surface loading acting normal to the shell in the outward direction. Its magnitude can vary in both the meridional and circumferential directions (e.g. under solids loading in a silo).

1.3.3.4 external pressure

Component of the surface loading acting normal to the shell in the inward direction. Its magnitude can vary in both the meridional and circumferential directions (e.g. under wind).

1.3.3.5 hydrostatic pressure

Pressure varying linearly with the axial coordinate of the shell of revolution.

1.3.3.6 wall friction load

Meridional component of the surface loading acting on the shell wall due to friction connected with internal pressure (e.g. when solids are contained within the shell).

1.3.3.7 local load

Point applied force or distributed load acting on a limited part of the circumference of the shell and over a limited height.

1.3.3.8 patch load

Local distributed load acting normal to the shell.

1.3.3.9 suction

Uniform net external pressure due to the reduced internal pressure in a shell with openings or vents under wind action.

1.3.3.10 partial vacuum

Uniform net external pressure due to the removal of stored liquids or solids from within a container that is inadequately vented.

1.3.3.11 thermal action

Temperature variation either down the shell meridian, or around the shell circumference or through the shell thickness.

1.3.4 Stress resultants and stresses in a shell

1.3.4.1 membrane stress resultants

The membrane stress resultants are the forces per unit width of shell that arise as the integral of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that is uniform through the shell thickness. There are three membrane stress resultants at any point (see figure 1.1(e)).

1.3.4.2 bending stress resultants

The bending stress resultants are the bending and twisting moments per unit width of shell that arise as the integral of the first moment of the distribution of direct and shear stresses acting parallel to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that varies linearly through the shell thickness, with value zero and the middle surface. There are two bending moments and one twisting moment at any point.

1.3.4.3 transverse shear stress resultants

The transverse stress resultants are the forces per unit width of shell that arise as the integral of the distribution of shear stresses acting normal to the shell middle surface through the thickness of the shell. Under elastic conditions, each of these stress resultants induces a stress state that varies parabolically through the shell thickness. There are two transverse shear stress resultants at any point (see figure 1.1(f)).

1.3.4.4 membrane stress

The membrane stress is defined as the membrane stress resultant divided by the shell thickness (see figure 1.1(e)).

1.3.4.5 bending stress

The bending stress is defined as the bending stress resultant multiplied by 6 and divided by the square of the shell thickness. It is only meaningful for conditions in which the shell is elastic.

1.3.5 Types of analysis

1.3.5.1 global analysis

An analysis that includes the complete structure, rather than individual structural parts treated separately.

1.3.5.2 membrane theory analysis

An analysis that predicts the behaviour of a thin-walled shell structure under distributed loads by assuming that only membrane forces satisfy equilibrium with the external loads.

1.3.5.3 linear elastic shell analysis (LA)

An analysis that predicts the behaviour of a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell.

1.3.5.4 linear elastic bifurcation (eigenvalue) analysis (LBA)

An analysis that evaluates the linear bifurcation eigenvalue for a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell. It should be noted that, where an eigenvalue is mentioned, this does not relate to vibration modes.

1.3.5.5 geometrically nonlinear elastic analysis (GNA)

An analysis based on the principles of shell bending theory applied to the perfect structure, using a linear elastic material law but including nonlinear large deflection theory for the displacements that accounts full for any change in geometry due to the actions on the shell. A bifurcation eigenvalue check is included at each load level.

1.3.5.6 materially nonlinear analysis (MNA)

An analysis based on shell bending theory applied to the perfect structure, using the assumption of small deflections, as in AC1) 1.3.5.3 (AC1, but adopting a nonlinear elasto-plastic material law.

1.3.5.7 geometrically and materially nonlinear analysis (GMNA)

An analysis based on shell bending theory applied to the perfect structure, using the assumptions of nonlinear large deflection theory for the displacements and a nonlinear elasto-plastic material law. A bifurcation eigenvalue check is included at each load level.

1.3.5.8 geometrically nonlinear elastic analysis with imperfections included (GNIA)

An analysis with imperfections explicitly included, similar to a GNA analysis as defined in AC1) 1.3.5.5 (AC1, but adopting a model for the geometry of the structure that includes the imperfect shape (i.e. the geometry of the middle surface includes unintended deviations from the ideal shape). The imperfection may also cover the effects of deviations in boundary conditions and / or the effects of residual stresses. A bifurcation eigenvalue check is included at each load level.

1.3.5.9 geometrically and materially nonlinear analysis with imperfections included (GMNIA)

An analysis with imperfections explicitly included, based on the principles of shell bending theory applied to the imperfect structure (i.e. the geometry of the middle surface includes unintended deviations from the ideal shape), including nonlinear large deflection theory for the displacements that accounts full for any change in geometry due to the actions on the shell and a nonlinear elasto-plastic material law. The imperfections may also include imperfections in boundary conditions and residual stresses. A bifurcation eigenvalue check is included at each load level.

1.3.6 Stress categories used in stress design

1.3.6.1 Primary stresses

The stress system required for equilibrium with the imposed loading. This consists primarily of membrane stresses, but in some conditions, bending stresses may also be required to achieve equilibrium.

1.3.6.2 Secondary stresses

Stresses induced by internal compatibility or by compatibility with the boundary conditions, associated with imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage). These stresses are not required to achieve equilibrium between an internal stress state and the external loading.

1.3.7 Special definitions for buckling calculations

1.3.7.1 critical buckling resistance

The smallest bifurcation or limit load determined assuming the idealised conditions of elastic material behaviour, perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses (LBA analysis).

1.3.7.2 critical buckling stress

The membrane stress associated with the critical buckling resistance.

1.3.7.3 plastic reference resistance

The plastic limit load, determined assuming the idealised conditions of rigid-plastic material behaviour, perfect geometry, perfect load application, perfect support and material isotropy (modelled using MNA analysis).

1.3.7.4 characteristic buckling resistance

The load associated with buckling in the presence of inelastic material behaviour, the geometrical and structural imperfections that are inevitable in practical construction, and follower load effects.

1.3.7.5 characteristic buckling stress

The membrane stress associated with the characteristic buckling resistance.

1.3.7.6 design buckling resistance

The design value of the buckling load, obtained by dividing the characteristic buckling resistance by the partial factor for resistance.

1.3.7.7 design buckling stress

The membrane stress associated with the design buckling resistance.

1.3.7.8 key value of the stress

The value of stress in a non-uniform stress field that is used to characterise the stress magnitudes in a buckling limit state assessment.

1.3.7.9 fabrication tolerance quality class

The category of fabrication tolerance requirements that is assumed in design, see 8.4.

1.4 Symbols

(1) In addition to those given in EN 1990 and EN 1993-1-1, the following symbols are used:

(2) Coordinate system, see figure 1.1:

- r radial coordinate, normal to the axis of revolution;
- x meridional coordinate;
- z axial coordinate;
- θ circumferential coordinate;
- ϕ meridional slope: angle between axis of revolution and normal to the meridian of the shell;

(3) Pressures:

- p_n normal to the shell;
- p_x meridional surface loading parallel to the shell;
- p_θ circumferential surface loading parallel to the shell;

(4) Line forces:

- P_n load per unit circumference normal to the shell;
- P_x load per unit circumference acting in the meridional direction;
- P_θ load per unit circumference acting circumferentially on the shell;

(5) Membrane stress resultants:

- n_x meridional membrane stress resultant;
- n_θ circumferential membrane stress resultant;
- $n_{x\theta}$ membrane shear stress resultant;

(6) Bending stress resultants:

- m_x meridional bending moment per unit width;

m_θ circumferential bending moment per unit width;

$m_{x\theta}$ twisting shear moment per unit width;

q_{xn} transverse shear force associated with meridional bending;

$q_{\theta n}$ transverse shear force associated with circumferential bending;

(7) Stresses:

σ_x meridional stress;

σ_θ circumferential stress;

σ_{eq} von Mises equivalent stress (can also take negative values during cyclic loading);

τ, $\tau_{x\theta}$ in-plane shear stress;

τ_{xn}, $\tau_{\theta n}$ meridional, circumferential transverse shear stresses associated with bending;

(8) Displacements:

u meridional displacement;
v circumferential displacement;
w displacement normal to the shell surface;
β_ϕ meridional rotation, see 5.2.2;

(9) Shell dimensions:

d internal diameter of shell;
L total length of the shell;
ℓ length of shell segment;
ℓ_g gauge length for measurement of imperfections;
$\ell_{g\theta}$ gauge length in circumferential direction for measurement of imperfections;
ℓ_{gw} gauge length across welds for measurement of imperfections;
ℓ_{gx} gauge length in meridional direction for measurement of imperfections;
ℓ_R limited length of shell for buckling strength assessment;
r radius of the middle surface, normal to the axis of revolution;
t thickness of shell wall;
t_{max} maximum thickness of shell wall at a joint;
t_{min} minimum thickness of shell wall at a joint;
t_{ave} average thickness of shell wall at a joint;
β apex half angle of cone;

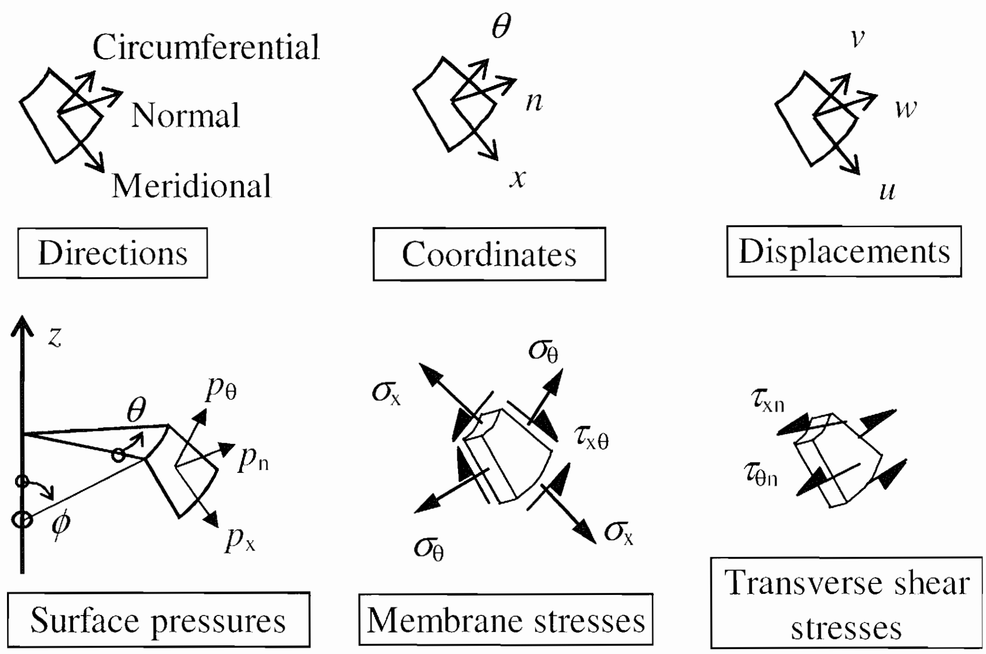


Figure 1.1: Symbols in shells of revolution

(10) Tolerances, see 8.4:

e	eccentricity between the middle surfaces of joined plates;
U_e	AC1 non-intended AC1 eccentricity tolerance parameter;
U_r	out-of-roundness tolerance parameter;
U_n	initial dimple imperfection amplitude parameter for numerical calculations;
U_0	initial dimple tolerance parameter;
Δw_0	tolerance normal to the shell surface;

(11) Properties of materials:

E	Young's modulus of elasticity;
f_{eq}	von Mises equivalent strength;
f_y	yield strength;
f_u	ultimate strength;
ν	Poisson's ratio;

(12) Parameters in strength assessment:

C	coefficient in buckling strength assessment;
D	cumulative damage in fatigue assessment;
F	generalised action;
F_{Ed}	action set on a complete structure corresponding to a design situation (design values);
F_{Rd}	calculated values of the action set at the maximum resistance condition of the structure (design values);
r_{Rk}	characteristic reference resistance ratio (used with subscripts to identify the basis): defined as

the ratio (F_{Rk} / F_{Ed});

r_{Rpl} plastic reference resistance ratio (defined as a load factor on design loads using MNA analysis);

r_{Rcr} critical buckling resistance ratio (defined as a load factor on design loads using LBA analysis);

NOTE: For consistency of symbols throughout the EN1993 the symbol for the reference resistance ratio r_{Ri} is used instead of the symbol R_{Ri}. However, in order to avoid misunderstanding, it needs to be noted here that the symbol R_{Ri} is widely used in the expert field of shell structure design.

k calibration factor for nonlinear analyses;

k power of interaction expressions in buckling strength interaction expressions;

n number of cycles of loading;

α elastic imperfection reduction factor in buckling strength assessment;

β plastic range factor in buckling interaction;

γ partial factor;

Δ range of parameter when alternating or cyclic actions are involved;

ε_p plastic strain;

η interaction exponent for buckling;

$\bar{\lambda}$ relative slenderness of shell;

$\bar{\lambda}_{ov}$ overall relative slenderness for the complete shell (multiple segments);

$\bar{\lambda}_0$ squash limit relative slenderness (value of $\bar{\lambda}$ above which resistance reductions due to instability or change of geometry occur);

$\bar{\lambda}_p$ plastic limit relative slenderness (value of $\bar{\lambda}$ below which plasticity affects the stability);

ω relative length parameter for shell;

χ buckling reduction factor for elastic-plastic effects in buckling strength assessment;

χ_{ov} overall buckling resistance reduction factor for complete shell;

(13) Subscripts:

E value of stress or displacement (arising from design actions);
F actions;
M material;
R resistance;
cr critical buckling value;
d design value;
int internal;
k characteristic value;
max maximum value;
min minimum value;
nom nominal value;
pl plastic value;

u ultimate;
y yield.

(14) Further symbols are defined where they first occur.

1.5 Sign conventions

(1) Outward direction positive: internal pressure positive, outward displacement positive, except as noted in (4).

(2) Tensile stresses positive, except as noted in (4).

NOTE: Compression is treated as positive in EN 1993-1-1.

(3) Shear stresses positive as shown in figures 1.1 and D.1.

(4) For simplicity, in section 8 and Annex D, compressive stresses are treated as positive. For these cases, both external pressures and internal pressures are treated as positive where they occur.

2 Basis of design and modelling

2.1 General

(1)P The basis of design shall be in accordance with EN 1990, as supplemented by the following.

(2) In particular, the shell should be designed in such a way that it will sustain all actions and satisfy the following requirements:

overall equilibrium;
equilibrium between actions and internal forces and moments, see sections 6 and 8;
limitation of cracks due to cyclic plastification, see section 7;
limitation of cracks due to fatigue, see section 9.

(3) The design of the shell should satisfy the serviceability requirements set out in the appropriate application standard (EN 1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3).

(4) The shell may be proportioned using design assisted by testing. Where appropriate, the requirements are set out in the appropriate application standard (EN 1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3).

(5) All actions should be introduced using their design values according to EN 1991 and EN 1993 Parts 3.1, 3.2, 4.1, 4.2, 4.3 as appropriate.

2.2 Types of analysis

2.2.1 General

(1) One or more of the following types of analysis should be used as detailed in section 4, depending on the limit state and other considerations:

Global analysis, see 2.2.2;
Membrane theory analysis, see 2.2.3;
Linear elastic shell analysis, see 2.2.4;
Linear elastic bifurcation analysis, see 2.2.5;
Geometrically nonlinear elastic analysis, see 2.2.6;
Materially nonlinear analysis, see 2.2.7;
Geometrically and materially nonlinear analysis, see 2.2.8;
Geometrically nonlinear elastic analysis with imperfections included, see 2.2.9;
Geometrically and materially nonlinear analysis with imperfections included, see 2.2.10.

2.2.2 Global analysis

(1) In a global analysis simplified treatments may be used for certain parts of the structure.

2.2.3 Membrane theory analysis

(1) A membrane theory analysis should only be used provided that the following conditions are met:

the boundary conditions are appropriate for transfer of the stresses in the shell into support reactions without causing significant bending effects;
the shell geometry varies smoothly in shape (without discontinuities);
the loads have a smooth distribution (without locally concentrated or point loads).

(2) A membrane theory analysis does not necessarily fulfil the compatibility of deformations at boundaries or between shell segments of different shape or between shell segments subjected to different loading. However, the resulting field of membrane forces satisfies the requirements of primary stresses (LS1).

2.2.4 Linear elastic shell analysis (LA)

(1) The linearity of the theory results from the assumptions of a linear elastic material law and the linear small deflection theory. Small deflection theory implies that the assumed geometry remains that of the undeformed structure.

(2) An LA analysis satisfies compatibility in the deformations as well as equilibrium. The resulting field of membrane and bending stresses satisfy the requirements of primary plus secondary stresses [AC1) (LS1 to LS4) (AC1].

2.2.5 Linear elastic bifurcation analysis (LBA)

(1) The conditions of 2.2.4 concerning the material and geometric assumptions are met. However, this linear bifurcation analysis obtains the lowest eigenvalue at which the shell may buckle into a different deformation mode, assuming no change of geometry, no change in the direction of action of the loads, and no material degradation. Imperfections of all kinds are ignored. This analysis provides the elastic critical buckling resistance r_{Rcr}, see 8.6 and 8.7 (LS3).

2.2.6 Geometrically nonlinear elastic analysis (GNA)

(1) A GNA analysis satisfies both equilibrium and compatibility of the deflections under conditions in which the change in the geometry of the structure caused by loading is included. The resulting field of stresses matches the definition of primary plus secondary stresses [AC1) (LS2 and LS4) (AC1].

(2) Where compression or shear stresses are predominant in some part of the shell, a GNA analysis delivers the elastic buckling load of the perfect structure, including changes in geometry, that may be of assistance in checking the limit state LS3, see 8.7.

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system must be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

2.2.7 Materially nonlinear analysis (MNA)

(1) The result of an MNA analysis gives the plastic limit load, which can be interpreted as a load amplification factor r_{Rpl} on the design value of the loads F_{Ed}. This analysis provides the plastic reference resistance ratio r_{Rpl} used in 8.6 and 8.7.

(2) An MNA analysis may be used to verify [AC1) limit states LS1 and LS3 (AC1].

(3) An MNA analysis may be used to give the plastic strain increment $\Delta\varepsilon$ during one cycle of cyclic loading that may be used to verify limit state LS2.

2.2.8 Geometrically and materially nonlinear analysis (GMNA)

(1) The result of a GMNA analysis, analogously to 2.2.7, gives the geometrically nonlinear plastic limit load of the perfect structure and the plastic strain increment, that may be used for checking the limit states LS1 and LS2.

(2) Where compression or shear stresses are predominant in some part of the shell, a GMNA analysis gives the elasto-plastic buckling load of the perfect structure, that may be of assistance in checking the limit state LS3, see 8.7.

(3) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

2.2.9 Geometrically nonlinear elastic analysis with imperfections included (GNIA)

(1) A GNIA analysis is used in cases where compression or shear stresses dominate in the shell. It delivers elastic buckling loads of the imperfect structure, that may be of assistance in checking the limit state LS3, see 8.7.

(2) Where this analysis is used for a buckling load evaluation (LS3), the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path. Care must be taken to ensure that the local stresses do not exceed values at which material nonlinearity may affect the behaviour.

2.2.10 Geometrically and materially nonlinear analysis with imperfections included (GMNIA)

(1) A GMNIA analysis is used in cases where compression or shear stresses are dominant in the shell. It delivers elasto-plastic buckling loads for the "real" imperfect structure, that may be used for checking the limit state LS3, see 8.7.

(2) Where this analysis is used for a buckling load evaluation, the eigenvalues of the system should be checked to ensure that the numerical process does not fail to detect a bifurcation in the load path.

(3) Where this analysis is used for a buckling load evaluation, an additional GMNA analysis of the perfect shell should always be conducted to ensure that the degree of imperfection sensitivity of the structural system is identified.

2.3 Shell boundary conditions

(1) The boundary conditions assumed in the design calculation should be chosen in such a way as to ensure that they achieve a realistic or conservative model of the real construction. Special attention should be given not only to the constraint of displacements normal to the shell wall (deflections), but also to the constraint of the displacements in the plane of the shell wall (meridional and circumferential) because of the significant effect these have on shell strength and buckling resistance.

(2) In shell buckling (eigenvalue) calculations (limit state LS3), the definition of the boundary conditions should refer to the incremental displacements during the buckling process, and not to total displacements induced by the applied actions before buckling.

(3) The boundary conditions at a continuously supported lower edge of a shell should take into account whether local uplifting of the shell is prevented or not.

(4) The shell edge rotation β_ϕ should be particularly considered in short shells and in the calculation of secondary stresses in longer shells (according to the limit states LS2 and LS4).

(5) The boundary conditions set out in 5.2.2 should be used in computer analyses and in selecting expressions from Annexes A to D.

(6) The structural connections between shell segments at a junction should be such as to ensure that the boundary condition assumptions used in the design of the individual shell segments are satisfied.

3 Materials and geometry

3.1 Material properties

(1) The material properties of steels should be obtained from the relevant application standard.

(2) Where materials with nonlinear stress-strain curves are involved and a buckling analysis is carried out under stress design (see 8.5), the initial tangent value of Young´s modulus E should be replaced by a reduced value. If no better method is available, the secant modulus at the 0,2% proof stress should be used when assessing the elastic critical load or elastic critical stress.

(3) In a global numerical analysis using material nonlinearity, the 0,2% proof stress should be used to represent the yield stress f_y in all relevant expressions. The stress-strain curve should be obtained from EN 1993-1-5 Annex C for carbon steels and EN 1993-1-4 Annex C for stainless steels.

(4) The material properties apply to temperatures not exceeding 150°C.

> **NOTE:** The national annex may give information about material properties at temperatures exceeding 150°C.

3.2 Design values of geometrical data

(1) The thickness t of the shell should be taken as defined in the relevant application standard. If no application standard is relevant, the nominal thickness of the wall, reduced by the prescribed value of the corrosion loss, should be used.

(2) The thickness ranges within which the rules of this Standard may be applied are defined in the relevant EN 1993 application parts.

(3) The middle surface of the shell should be taken as the reference surface for loads.

(4) The radius r of the shell should be taken as the nominal radius of the middle surface of the shell, measured normal to the axis of revolution.

(5) The buckling design rules of this Standard should not be applied outside the ranges of the r/t ratio set out in section 8 or Annex D or in the relevant EN 1993 application parts.

3.3 Geometrical tolerances and geometrical imperfections

(1) Tolerance values for the deviations of the geometry of the shell surface from the nominal values are defined in the execution standards due to the requirements of serviceability. Relevant items are:

out-of-roundness (deviation from circularity),
eccentricities (deviations from a continuous middle surface in the direction normal to the shell across the junctions between plates),
local dimples (local normal deviations from the nominal middle surface).

> **NOTE:** The requirements for execution are set out in EN 1090, but a fuller description of these tolerances is given here because of the critical relationship between the form of the tolerance measure, its amplitude and the evaluated resistance of the shell structure.

(2) If the limit state of buckling (LS3, as described in 4.1.3) is one of the ultimate limit states to be considered, additional buckling-relevant geometrical tolerances have to be observed in order to keep the geometrical imperfections within specified limits. These buckling-relevant geometrical tolerances are quantified in section 8 or in the relevant EN 1993 application parts.

(3) Calculation values for the deviations of the shell surface geometry from the nominal geometry, as required for geometrical imperfection assumptions (overall imperfections or local imperfections) for the buckling design by global GMNIA analysis (see 8.7), should be derived from the specified geometrical tolerances. Relevant rules are given in 8.7 or in relevant EN 1993 application parts.

4 Ultimate limit states in steel shells

4.1 Ultimate limit states to be considered

4.1.1 LS1: Plastic limit

(1) The limit state of the plastic limit should be taken as the condition in which the capacity of the structure to resist the actions on it is exhausted by yielding of the material. The resistance offered by the structure at the plastic limit state may be derived as the plastic collapse load obtained from a mechanism based on small displacement theory.

(2) The limit state of tensile rupture should be taken as the condition in which the shell wall experiences gross section tensile failure, leading to separation of the two parts of the shell.

(3) In the absence of fastener holes, verification at the limit state of tensile rupture may be assumed to be covered by the check for the plastic limit state. However, where holes for fasteners occur, a supplementary check in accordance with 6.2 of EN 1993-1-1 should be carried out.

(4) In verifying the plastic limit state, plastic or partially plastic behaviour of the structure may be assumed (i.e. elastic compatibility considerations may be neglected).

> **NOTE:** The basic characteristic of this limit state is that the load or actions sustained (resistance) cannot be increased without exploiting a significant change in the geometry of the structure or strain-hardening of the material.

(5) All relevant load combinations should be accounted for when checking LS1.

(6) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS1:

membrane theory;
expressions in Annexes A and B;
linear elastic analysis (LA);
materially nonlinear analysis (MNA);
geometrically and materially nonlinear analysis (GMNA).

4.1.2 LS2: Cyclic plasticity

(1) The limit state of cyclic plasticity should be taken as the condition in which repeated cycles of loading and unloading produce yielding in tension and in compression at the same point, thus causing plastic work to be repeatedly done on the structure, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material.

> **NOTE:** The stresses that are associated with this limit state develop under a combination of all actions and the compatibility conditions for the structure.

(2) All variable actions (such as imposed loads and temperature variations) that can lead to yielding, and which might be applied with more than three cycles in the life of the structure, should be accounted for when checking LS2.

(3) In the verification of this limit state, compatibility of the deformations under elastic or elastic-plastic conditions should be considered.

(4) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS2:

expressions in Annex C;
elastic analysis (LA or GNA);
MNA or GMNA to determine the plastic strain range.

(5) Low cycle fatigue failure may be assumed to be prevented if the procedures set out in this standard are adopted.

4.1.3 LS3: Buckling

(1) The limit state of buckling should be taken as the condition in which all or part of the structure suddenly develops large displacements normal to the shell surface, caused by loss of stability under compressive membrane or shear membrane stresses in the shell wall, leading to inability to sustain any increase in the stress resultants, possibly causing total collapse of the structure.

(2) One or more of the following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS3:

membrane theory for axisymmetric conditions only (for exceptions, see relevant application parts of EN 1993)
expressions in Annex A;
linear elastic analysis (LA), which is a minimum requirement for stress analysis under general loading conditions (unless the load case is given in Annex A);
linear elastic bifurcation analysis (LBA), which is required for shells under general loading conditions if the critical buckling resistance is to be used;
materially nonlinear analysis (MNA), which is required for shells under general loading conditions if the reference plastic resistance is to be used;
GMNIA, coupled with MNA, LBA and GMNA, using appropriate imperfections and calculated calibration factors.

(3) All relevant load combinations causing compressive membrane or shear membrane stresses in the shell should be accounted for when checking LS3.

(4) Because the strength under limit state LS3 depends strongly on the quality of construction, the strength assessment should take account of the associated requirements for execution tolerances.

NOTE: For this purpose, three classes of geometrical tolerances, termed "fabrication quality classes" are given in section 8.

4.1.4 LS4: Fatigue

(1) The limit state of fatigue should be taken as the condition in which repeated cycles of increasing and decreasing stress lead to the development of a fatigue crack.

(2) The following methods of analysis (see 2.2) should be used for the calculation of the design stresses and stress resultants when checking LS4:

expressions in Annex C, using stress concentration factors;
elastic analysis (LA or GNA), using stress concentration factors.

(3) All variable actions that will be applied with more than N_f cycles in the design life time of the structure according to the relevant action spectrum in EN 1991 in accordance with the appropriate application part of EN 1993-3 or EN 1993-4, should be accounted for when checking LS4.

NOTE: The National Annex may choose the value of N_f. The value $N_f = 10\,000$ is recommended.

4.2 Design concepts for the limit states design of shells

4.2.1 General

(1) The limit state verification should be carried out using one of the following:

stress design;
direct design by application of standard expressions;
design by global numerical analysis (for example, by means of computer programs such as those based on the finite element method).

(2) Account should be taken of the fact that elasto-plastic material responses induced by different stress components in the shell have different effects on the failure modes and the ultimate limit states. The stress components should therefore be placed in stress categories with different limits. Stresses that develop to meet equilibrium requirements should be treated as more significant than stresses that are induced by the compatibility of deformations normal to the shell. Local stresses caused by notch effects in construction details may be assumed to have a negligibly small influence on the resistance to static loading.

(3) The categories distinguished in the stress design should be primary, secondary and local stresses. Primary and secondary stress states may be replaced by stress resultants where appropriate.

(4) In a global analysis, the primary and secondary stress states should be replaced by the limit load and the strain range for cyclic loading.

(5) In general, it may be assumed that primary stress states control LS1, LS3 depends strongly on primary stress states but may be affected by secondary stress states, LS2 depends on the combination of primary and secondary stress states, and local stresses govern LS4.

4.2.2 Stress design

4.2.2.1 General

(1) Where the stress design approach is used, the limit states should be assessed in terms of three categories of stress: primary, secondary and local. The categorisation is performed, in general, on the von Mises equivalent stress at a point, but buckling stresses cannot be assessed using this value.

4.2.2.2 Primary stresses

(1) The primary stresses should be taken as the stress system required for equilibrium with the imposed loading. They may be calculated from any realistic statically admissible determinate system. The plastic limit state (LS1) should be deemed to be reached when the primary stress reaches the yield strength throughout the full thickness of the wall at a sufficient number of points, such that only the strain hardening reserve or a change of geometry would lead to an increase in the resistance of the structure.

(2) The calculation of primary stresses should be based on any system of stress resultants, consistent with the requirements of equilibrium of the structure. It may also take into account the benefits of plasticity theory. Alternatively, since linear elastic analysis satisfies equilibrium requirements, its predictions may also be used as a safe representation of the plastic limit state (LS1). Any of the analysis methods given in 5.3 may be applied.

(3) Because limit state design for LS1 allows for full plastification of the cross-section, the primary stresses due to bending moments may be calculated on the basis of the plastic section modulus, see 6.2.1. Where there is interaction between stress resultants in the cross-section, interaction rules based on the von Mises yield criterion may be applied.

(4) The primary stresses should be limited to the design value of the yield strength, see section 6 (LS1).

4.2.2.3 Secondary stresses

(1) In statically indeterminate structures, account should be taken of the secondary stresses, induced by internal compatibility and compatibility with the boundary conditions that are caused by imposed loading or imposed displacements (temperature, prestressing, settlement, shrinkage).

NOTE: As the von Mises yield condition is approached, the displacements of the structure increase without further increase in the stress state.

(2) Where cyclic loading causes plasticity, and several loading cycles occur, consideration should be given to the possible reduction of resistance caused by the secondary stresses. Where the cyclic loading is of such a magnitude that yielding occurs both at the maximum load and again on unloading, account should be taken of a possible failure by cyclic plasticity associated with the secondary stresses.

(3) If the stress calculation is carried out using a linear elastic analysis that allows for all relevant compatibility conditions (effects at boundaries, junctions, variations in wall thickness etc.), the stresses that vary linearly through the thickness may be taken as the sum of the primary and secondary stresses and used in an assessment involving the von Mises yield criterion, see 6.2.

NOTE: The secondary stresses are never needed separately from the primary stresses.

(4) The secondary stresses should be limited as follows:

The sum of the primary and secondary stresses (including bending stresses) should be limited to $2f_{yd}$ for the condition of cyclic plasticity (LS2: see section 7);

The membrane component of the sum of the primary and secondary stresses should be limited by the design buckling resistance (LS3: see section 8).

The sum of the primary and secondary stresses (including bending stresses) should be limited to the fatigue resistance (LS4: see section 9).

4.2.2.4 Local stresses

(1) The highly localised stresses associated with stress raisers in the shell wall due to notch effects (holes, welds, stepped walls, attachments, and joints) should be taken into account in a fatigue assessment (LS4).

(2) For construction details given in EN 1993-1-9, the fatigue design may be based on the nominal linear elastic stresses (sum of the primary and secondary stresses) at the relevant point. For all other details, the local stresses may be calculated by applying stress concentration factors (notch factors) to the stresses calculated using a linear elastic stress analysis.

(3) The local stresses should be limited according to the requirements for fatigue (LS4) set out in section 9.

4.2.3 Direct design

(1) Where direct design is used, the limit states may be represented by standard expressions that have been derived from either membrane theory, plastic mechanism theory or linear elastic analysis.

(2) The membrane theory expressions given in Annex A may be used to determine the primary stresses needed for assessing LS1 and LS3.

(3) The expressions for plastic design given in Annex B may be used to determine the plastic limit loads needed for assessing LS1.

(4) The expressions for linear elastic analysis given in Annex C may be used to determine stresses of the primary plus secondary stress type needed for assessing LS2 and LS4. An LS3 assessment may be based on the membrane part of these expressions.

4.2.4 Design by global numerical analysis

(1) Where a global numerical analysis is used, the assessment of the limit states should be carried out using one of the alternative types of analysis specified in 2.2 (but not membrane theory analysis) applied to the complete structure.

(2) Linear elastic analysis (LA) may be used to determine stresses or stress resultants, for use in assessing LS2 and LS4. The membrane parts of the stresses found by LA may be used in assessing LS3. LS1 may be assessed using LA, but LA only gives an approximate estimate and its results should be interpreted as set out in section 6.

(3) Linear elastic bifurcation analysis (LBA) may be used to determine the critical buckling resistance of the structure, for use in assessing LS3.

(4) A materially nonlinear analysis (MNA) may be used to determine the plastic reference resistance, and this may be used for assessing LS1. Under a cyclic loading history, an MNA analysis may be used to determine plastic strain incremental changes, for use in assessing LS2. The plastic reference resistance is also required as part of the assessment of LS3, and this may be found from an MNA analysis.

(5) Geometrically nonlinear elastic analyses (GNA and GNIA) include consideration of the deformations of the structure, but none of the design methodologies of section 8 permit these to be used without a GMNIA analysis. A GNA analysis may be used to determine the elastic buckling load of the perfect structure. A GNIA analysis may be used to determine the elastic buckling load of the imperfect structure.

(6) Geometrically and materially nonlinear analysis (GMNA and GMNIA) may be used to determine collapse loads for the perfect (GMNA) and the imperfect structure (GMNIA). The GMNA analysis may be used in assessing LS1, as detailed in 6.3. The GMNIA collapse load may be used, with additional consideration of the GMNA collapse load, for assessing LS3 as detailed in 8.7. Under a cyclic loading history, the plastic strain incremental changes taken from a GMNA analysis may be used for assessing LS2.

5 Stress resultants and stresses in shells

5.1 Stress resultants in the shell

(1) In principle, the eight stress resultants in the shell wall at any point should be calculated and the assessment of the shell with respect to each limit state should take all of them into account. However, the shear stresses τ_{xn}, $\tau_{\theta n}$ due to the transverse shear forces q_{xn}, $q_{\theta n}$ are insignificant compared with the other components of stress in almost all practical cases, so they may usually be neglected in design.

(2) Accordingly, for most design purposes, the evaluation of the limit states may be made using only the six stress resultants in the shell wall n_x, n_θ, $n_{x\theta}$, m_x, m_θ, $m_{x\theta}$. Where the structure is axisymmetric and subject only to axisymmetric loading and support, only n_x, n_θ, m_x and m_θ need be used.

(3) If any uncertainty arises concerning the stress to be used in any of the limit state verifications, the von Mises equivalent stress on the shell surface should be used.

5.2 Modelling of the shell for analysis

5.2.1 Geometry

(1) The shell should be represented by its middle surface.

(2) The radius of curvature should be taken as the nominal radius of curvature. Imperfections should be neglected, except as set out in section 8 (LS3 buckling limit state).

(3) An assembly of shell segments should not be subdivided into separate segments for analysis unless the boundary conditions for each segment are chosen in such a way as to represent interactions between them in a conservative manner.

(4) A base ring intended to transfer local support forces into the shell should not be separated from the shell it supports in an assessment of limit state LS3.

(5) Eccentricities and steps in the shell middle surface should be included in the analysis model if they induce significant bending effects as a result of the membrane stress resultants following an eccentric path.

(6) At junctions between shell segments, any eccentricity between the middle surfaces of the shell segments should be considered in the modelling.

(7) A ring stiffener should be treated as a separate structural component of the shell, except where the spacing of the rings is closer than $1{,}5\sqrt{rt}$.

(8) A shell that has discrete stringer stiffeners attached to it may be treated as an orthotropic uniform shell, provided that the stringer stiffeners are no further apart than $5\sqrt{rt}$.

(9) A shell that is corrugated (vertically or horizontally) may be treated as an orthotropic uniform shell provided that the corrugation wavelength is less than $0{,}5\sqrt{rt}$.

(10) A hole in the shell may be neglected in the modelling provided its largest dimension is smaller than $0{,}5\sqrt{rt}$.

(11) The overall stability of the complete structure should be verified as detailed in EN 1993 Parts 3.1, 3.2, 4.1, 4.2 or 4.3 as appropriate.

5.2.2 Boundary conditions

(1) The appropriate boundary conditions should be used in analyses for the assessment of limit states according to the conditions shown in table 5.1. For the special conditions needed for buckling calculations, reference should be made to 8.3.

(2) Rotational restraints at shell boundaries may be neglected in modelling for limit state LS1, but should be included in modelling for limit states LS2 and LS4. For short shells (see Annex D), the rotational restraint should be included for limit state LS3.

(3) Support boundary conditions should be checked to ensure that they do not cause excessive non-uniformity of transmitted forces or introduced forces that are eccentric to the shell middle surface. Reference should be made to the relevant EN 1993 application parts for the detailed application of this rule to silos and tanks.

(4) When a global numerical analysis is used, the boundary condition for the normal displacement w should also be used for the circumferential displacement v, except where special circumstances make this inappropriate.

Table 5.1: Boundary conditions for shells

Boundary condition code	Simple term	Description	Normal displacement s	Meridional displacements	Meridional rotation
BC1r	Clamped	radially restrained meridionally restrained rotation restrained	$w = 0$	$u = 0$	$\beta_\phi = 0$
BC1f		radially restrained meridionally restrained rotation free	$w = 0$	$u = 0$	$\beta_\phi \neq 0$
BC2r		radially restrained meridionally free rotation restrained	$w = 0$	$u \neq 0$	$\beta_\phi = 0$
BC2f	Pinned	radially restrained meridionally free rotation free	$w = 0$	$u \neq 0$	$\beta_\phi \neq 0$
BC3	Free edge	radially free meridionally free rotation free	$w \neq 0$	$u \neq 0$	$\beta_\phi \neq 0$

NOTE: The circumferential displacement v is closely linked to the displacement w normal to the surface, so separate boundary conditions are not identified for these two parameters (see (4)) but the values in column 4 should be adopted for displacement v.

5.2.3 Actions and environmental influences

(1) Actions should all be assumed to act at the shell middle surface. Eccentricities of load should be represented by static equivalent forces and moments at the shell middle surface.

(2) Local actions and local patches of action should not be represented by equivalent uniform loads except as detailed in section 8 for buckling (LS3).

(3) The modelling should account for whichever of the following are relevant:

local settlement under shell walls;
local settlement under discrete supports;
uniformity / non-uniformity of support of structure;
thermal differentials from one side of the structure to the other;
thermal differentials from inside to outside the structure;
wind effects on openings and penetrations;
interaction of wind effects on groups of structures;
connections to other structures;
conditions during erection.

5.2.4 Stress resultants and stresses

(1) Provided that the radius to thickness ratio is greater than $(r/t)_{min}$, the curvature of the shell may be ignored when calculating the stress resultants from the stresses in the shell wall.

> **NOTE:** The National Annex may choose the value of $(r/t)_{min}$. The value $(r/t)_{min} = 25$ is recommended.

5.3 Types of analysis

(1) The design should be based on one or more of the types of analysis given in table 5.2. Reference should be made to 2.2 for the conditions governing the use of each type of analysis.

Table 5.2: Types of shell analysis

Type of analysis	Shell theory	Material law	Shell geometry
Membrane theory of shells	membrane equilibrium	not applicable	perfect
Linear elastic shell analysis (LA)	linear bending and stretching	linear	perfect
Linear elastic bifurcation analysis (LBA)	linear bending and stretching	linear	perfect
Geometrically non-linear elastic analysis (GNA)	non-linear	linear	perfect
Materially non-linear analysis (MNA)	linear	non-linear	perfect
Geometrically and materially non-linear analysis (GMNA)	non-linear	non-linear	perfect
Geometrically non-linear elastic analysis with imperfections (GNIA)	non-linear	linear	imperfect
Geometrically and materially non-linear analysis with imperfections (GMNIA)	non-linear	non-linear	imperfect

6 Plastic limit state (LS1)

6.1 Design values of actions

(1)P The design values of the actions shall be based on the most adverse relevant load combination (including the relevant γ_F and ψ factors).

(2) Only those actions that represent loads affecting the equilibrium of the structure need be included.

6.2 Stress design

6.2.1 Design values of stresses

(1) Although stress design is based on an elastic analysis and therefore cannot accurately predict the plastic limit state, it may be used, on the basis of the lower bound theorem, to provide a conservative assessment of the plastic collapse resistance which is used to represent the plastic limit state, see 4.1.1.

(2) The Ilyushin yield criterion may be used, as detailed in (6), that comes closer to the true plastic collapse state than a simple elastic surface stress evaluation.

(3) At each point in the structure the design value of the stress $\sigma_{eq,Ed}$ should be taken as the highest primary stress determined in a structural analysis that considers the laws of equilibrium between imposed design load and internal forces and moments.

(4) The primary stress may be taken as the maximum value of the stresses required for equilibrium with the applied loads at a point or along an axisymmetric line in the shell structure.

(5) Where a membrane theory analysis is used, the resulting two-dimensional field of stress resultants $n_{x,Ed}$, $n_{\theta,Ed}$ and $n_{x\theta,Ed}$ may be represented by the equivalent design stress $\sigma_{eq,Ed}$ obtained from:

$$\sigma_{eq,Ed} = \frac{1}{t}\sqrt{n_{x,Ed}^2 + n_{\theta,Ed}^2 - n_{x,Ed} \cdot n_{\theta,Ed} + 3n_{x\theta,Ed}^2} \quad \text{... (6.1)}$$

(6) Where an LA or GNA analysis is used, the resulting two dimensional field of primary stresses may be represented by the von Mises equivalent design stress:

$$\sigma_{eq,Ed} = \sqrt{\sigma_{x,Ed}^2 + \sigma_{\theta,Ed}^2 - \sigma_{x,Ed} \cdot \sigma_{\theta,Ed} + 3\left(\tau_{x,\theta,Ed}^2 + \tau_{xn,Ed}^2 + \tau_{\theta n,Ed}^2\right)} \quad \text{... (6.2)}$$

in which:

$$\sigma_{x,Ed} = \frac{n_{x,Ed}}{t} \pm \frac{m_{x,Ed}}{(t^2/4)}, \qquad \sigma_{\theta,Ed} = \frac{n_{\theta,Ed}}{t} \pm \frac{m_{\theta,Ed}}{(t^2/4)} \quad \text{... (6.3)}$$

$$\tau_{x\theta,Ed} = \frac{n_{x\theta,Ed}}{t} \pm \frac{m_{x\theta,Ed}}{(t^2/4)}, \qquad \tau_{xn,Ed} = \frac{q_{xn,Ed}}{t}, \qquad \tau_{\theta n,Ed} = \frac{q_{\theta n,Ed}}{t} \quad \text{... (6.4)}$$

NOTE 1: The above expressions give a simplified conservative equivalent stress for design purposes.

NOTE2: The values of $\tau_{xn,Ed}$ and $\tau_{\theta n,Ed}$ are usually very small and do not affect the plastic resistance, so they may generally be ignored.

6.2.2 Design values of resistance

(1) The von Mises design strength should be taken from:

$$f_{eq,Rd} = f_{yd} = f_{yk} / \gamma_{M0} \quad \text{... (6.5)}$$

(2) The partial factor for resistance γ_{M0} should be taken from the relevant application standard.

(3) Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{M0}, the value of γ_{M0} should be taken from EN1993-1-1.

(4) Where the material has a nonlinear stress strain curve, the value of the characteristic yield strength f_{yk} should be taken as the 0,2% proof stress.

(5) The effect of fastener holes should be taken into account in accordance with 6.2.3 of EN1993-1-1 for tension and 6.2.4 of EN1993-1-1 for compression. For the tension check, the resistance should be based on the design value of the ultimate strength f_{ud}.

6.2.3 Stress limitation

(1)P In every verification of this limit state, the design stresses shall satisfy the condition:

$$\sigma_{eq,Ed} \leq f_{eq,Rd} \quad \text{... (6.6)}$$

6.3 Design by global numerical MNA or GMNA analysis

(1)P The design plastic limit resistance shall be determined as a load factor r_R applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The design values of the actions F_{Ed} should be determined as detailed in 6.1. The relevant load cases should be formed according to the required load combinations.

(3) In an MNA or GMNA analysis based on the design yield strength f_{yd}, the shell should be subject to the design values of the load cases detailed in (2), progressively increased by the load ratio r_R until the plastic limit condition is reached.

(4) Where an MNA analysis is used, the load ratio $r_{R.MNA}$ may be taken as the largest value attained in the analysis, ignoring the effect of strain hardening. This load ratio is identified as the plastic reference resistance ratio r_{Rpl} in 8.7.

(5) Where a GMNA analysis is used, if the analysis predicts a maximum load followed by a descending path, the maximum value should be used to determine the load ratio $r_{R.GMNA}$. Where a GMNA analysis does not predict a maximum load, but produces a progressively rising action-displacement relationship without strain hardening of the material, the load ratio $r_{R.GMNA}$ should be taken as no larger than the value at which the maximum von Mises equivalent plastic strain in the structure attains the value $\varepsilon_{mps} = n_{mps} (f_{yd} / E)$.

NOTE: The National Annex may choose the value of n_{mps}. The value $n_{mps} = 50$ is recommended.

(6) The characteristic plastic limit resistance r_{Rk} should be taken as either $r_{R.MNA}$ or $r_{R.GMNA}$ according to the analysis that has been used.

(7)P The design plastic limit resistance F_{Rd} shall be obtained from:

$$F_{Rd} = F_{Rk} / \gamma_{M0} = \frac{r_{Rk} \cdot F_{Ed}}{\gamma_{M0}} = r_{Rd} \cdot F_{Ed} \qquad \text{... (6.7)}$$

where:

γ_{M0} is the partial factor for resistance to plasticity according to 6.2.2.

(8)P It shall be verified that:

$$F_{Ed} \leq F_{Rd} = r_{Rd} \cdot F_{Ed} \quad \text{or} \quad r_{Rd} \geq 1 \qquad \text{... (6.8)}$$

6.4 Direct design

(1) For each shell segment in the structure represented by a basic loading case as given by Annex A, the highest von Mises membrane stress $\sigma_{eq.Ed}$ determined under the design values of the actions F_{Ed} should be limited to the stress resistance according to 6.2.2.

(2) For each shell or plate segment in the structure represented by a basic load case as given in Annex B, the design value of the actions F_{Ed} should not exceed the resistance F_{Rd} based on the design yield strength f_{yd}.

(3) Where net section failure at a bolted joint is a design criterion, the design value of the actions F_{Ed} should be determined for each joint. Where the stress can be represented by a basic load case as given in Annex A, and where the resulting stress state involves only membrane stresses, F_{Ed} should not exceed the resistance F_{Rd} based on the design ultimate strength f_{ud}, see 6.2.2(5).

7 Cyclic plasticity limit state (LS2)

7.1 Design values of actions

(1) Unless an improved definition is used, the design values of the actions for each load case should be chosen as the characteristic values of those parts of the total actions that are expected to be applied and removed more than three times in the design life of the structure.

(2) Where an elastic analysis or the expressions from Annex C are used, only the varying part of the actions between the extreme upper and lower values should be taken into account.

(3) Where a materially nonlinear computer analysis is used, the varying part of the actions between the extreme upper and lower values should be considered to act in the presence of coexistent permanent parts of the load.

7.2 Stress design

7.2.1 Design values of stress range

(1) The shell should be analysed using an LA or GNA analysis of the structure subject to the two extreme design values of the actions F_{Ed}. For each extreme load condition in the cyclic process, the stress components should be evaluated. From adjacent extremes in the cyclic process, the design values of the change in each stress component $\Delta\sigma_{x,Ed,i}$, $\Delta\sigma_{\theta,Ed,i}$, $\Delta\tau_{x\theta,Ed,i}$ on each shell surface (represented as i=1,2 for the inner and outer surfaces of the shell) and at any point in the structure should be determined. From these changes in stress, the design value of the von Mises equivalent stress change on the inner and outer surfaces should be found from:

$$\Delta\sigma_{eq,Ed,i} = \sqrt{\Delta\sigma^2_{x,Ed,i} - \Delta\sigma_{x,Ed,i} \cdot \Delta\sigma_{\theta,Ed,i} + \Delta\sigma^2_{\theta,Ed} + 3\Delta\tau^2_{x\,\theta,Ed,i}} \quad \text{... (7.1)}$$

(2) The design value of the stress range $\Delta\sigma_{eq,Ed}$ should be taken as the largest change in the von Mises equivalent stress changes $\Delta\sigma_{eq,Ed,i}$, considering each shell surface in turn (i=1 and i=2 considered separately).

(3) At a junction between shell segments, where the analysis models the intersection of the middle surfaces and ignores the finite size of the junction, the stress range may be taken at the first physical point in the shell segment (as opposed to the value calculated at the intersection of the two middle surfaces).

NOTE: This allowance is relevant where the stress changes very rapidly close to the junction.

7.2.2 Design values of resistance

(1) The von Mises equivalent stress range resistance $\Delta f_{eq,Rd}$ should be determined from:

$$\Delta f_{eq,Rd} = 2 f_{yd} \quad \text{... (7.2)}$$

7.2.3 Stress range limitation

(1)P In every verification of this limit state, the design stress range shall satisfy:

$$\Delta\sigma_{eq,Ed} \leq \Delta f_{eq,Rd} \quad \text{... (7.3)}$$

7.3 Design by global numerical MNA or GMNA analysis

7.3.1 Design values of total accumulated plastic strain

(1) Where a materially nonlinear global numerical analysis (MNA or GMNA) is used, the shell should be subject to the design values of the varying and permanent actions detailed in 7.1.

NOTE 1: It is usual to use an MNA analysis for this purpose.

NOTE 2: The National Annex may give recommendations for a more refined analysis.

(2) The total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ at the end of the design life of the structure should be assessed.

(3) The total accumulated von Mises equivalent plastic strain may be determined using an analysis that models all cycles of loading during the design life.

(4) Unless a more refined analysis is carried out, the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ may be determined from:

$$\varepsilon_{p,eq,Ed} = n\,\Delta\varepsilon_{p,eq,Ed} \qquad \text{... (7.4)}$$

where:

n is the number of cycles of loading in the design life of the structure;

$\Delta\varepsilon_{p,eq,Ed}$ is the largest increment in the von Mises equivalent plastic strain during one complete load cycle at any point in the structure, occurring after the third cycle.

(5) It may be assumed that "at any point in the structure" means at any point not closer to a notch or local discontinuity than the thickest adjacent plate thickness.

7.3.2 Total accumulated plastic strain limitation

(1) Unless a more sophisticated low cycle fatigue assessment is undertaken, the design value of the total accumulated von Mises equivalent plastic strain $\varepsilon_{p,eq,Ed}$ should satisfy the condition:

$$\varepsilon_{p,eq,Ed} \leq n_{p,eq}\,(f_{yd}\,/\,E) \qquad \text{... (7.5)}$$

NOTE: The National Annex may choose the value of $n_{p,eq}$. The value $n_{p,eq} = 25$ is recommended.

7.4 Direct design

(1) For each shell segment in the structure, represented by a basic loading case as given by Annex C, the highest von Mises equivalent stress range $\Delta\sigma_{eq,Ed}$ considering both shell surfaces under the design values of the actions F_{Ed} should be determined using the relevant expressions given in Annex C. The further assessment procedure should be as detailed in 7.2.

8 Buckling limit state (LS3)

8.1 Design values of actions

(1)P All relevant combinations of actions causing compressive membrane stresses or shear membrane stresses in the shell wall shall be taken into account.

8.2 Special definitions and symbols

(1) Reference should be made to the special definitions of terms concerning buckling in 1.3.6.

(2) In addition to the symbols defined in 1.4, additional symbols should be used in this section 8 as set out in (3) and (4).

(3) The stress resultant and stress quantities should be taken as follows:

$n_{x,Ed}$, $\sigma_{x,Ed}$ are the design values of the acting buckling-relevant meridional membrane stress resultant and stress (positive when compression);

$n_{\theta,Ed}$, $\sigma_{\theta,Ed}$ are the design values of the acting buckling-relevant circumferential membrane (hoop) stress resultant and stress (positive when compression);

$n_{x\theta,Ed}$, $\tau_{x\theta,Ed}$ are the design values of the acting buckling-relevant shear membrane stress resultant and stress.

(4) Buckling resistance parameters for use in stress design:

$\sigma_{x,Rcr}$ is the meridional elastic critical buckling stress;

$\sigma_{\theta,Rcr}$ is the circumferential elastic critical buckling stress;

$\tau_{x\theta,Rcr}$ is the shear elastic critical buckling stress;

$\sigma_{x,Rk}$ is the meridional characteristic buckling stress;

$\sigma_{\theta,Rk}$ is the circumferential characteristic buckling stress;

$\tau_{x\theta,Rk}$ is the shear characteristic buckling stress;

$\sigma_{x,Rd}$ is the meridional design buckling stress;

$\sigma_{\theta,Rd}$ is the circumferential design buckling stress;

$\tau_{x\theta,Rd}$ is the shear design buckling stress.

NOTE: This is a special convention for shell design that differs from that detailed in EN1993-1-1.

(5) The sign convention for use with LS3 should be taken as compression positive for meridional and circumferential stresses and stress resultants.

8.3 Buckling-relevant boundary conditions

(1) For the buckling limit state, special attention should be paid to the boundary conditions which are relevant to the incremental displacements of buckling (as opposed to pre-buckling displacements). Examples of relevant boundary conditions are shown in figure 8.1, in which the codes of table 5.1 are used.

8.4 Buckling-relevant geometrical tolerances

8.4.1 General

(1) Unless specific buckling-relevant geometrical tolerances are given in the relevant EN 1993 application parts, the following tolerance limits should be observed if LS3 is one of the ultimate limit states to be considered.

NOTE 1: The characteristic buckling stresses determined hereafter include imperfections that are based on the amplitudes and forms of geometric tolerances that are expected to be met during execution.

NOTE 2: The geometric tolerances given here are those that are known to have a large impact on the safety of the structure.

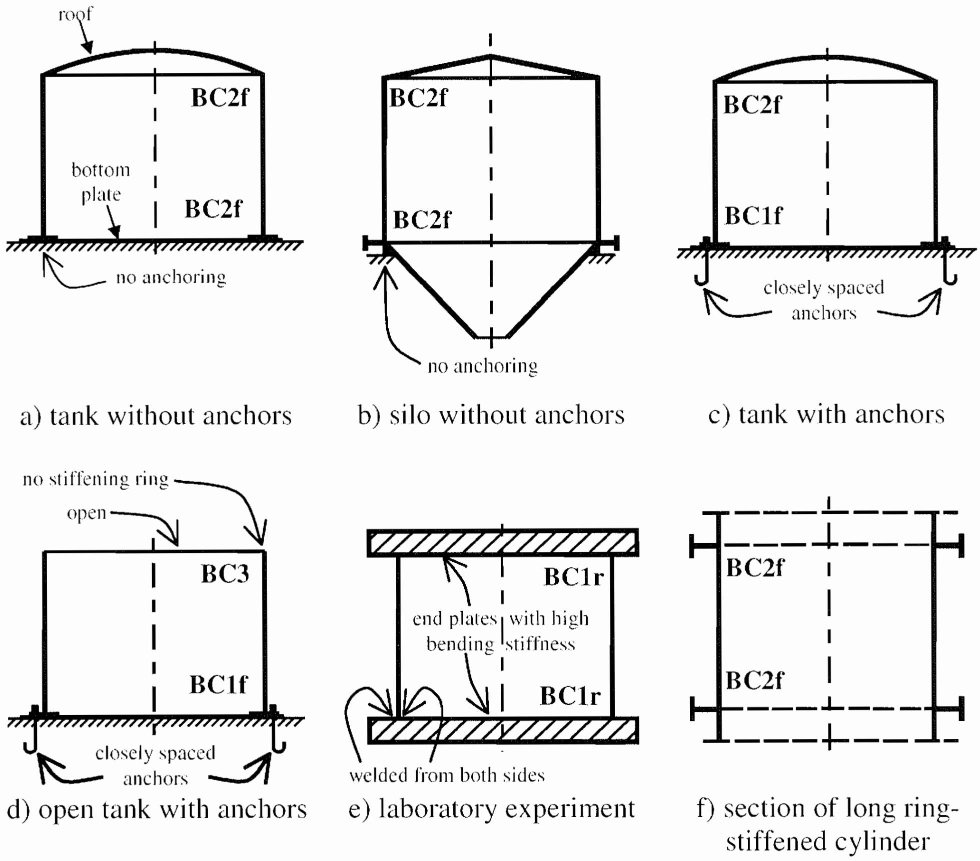


Figure 8.1: Schematic examples of boundary conditions for limit state LS3

(2) The fabrication tolerance quality class should be chosen as Class A, Class B or Class C according to the tolerance definitions in 8.4.2, 8.4.3, 8.4.4 and 8.4.5. The description of each class relates only to the strength evaluation.

> **NOTE:** The tolerances defined here match those specified in the execution standard EN 1090, but are set out more fully here to give the detail of the relationship between the imperfection amplitudes and the evaluated resistance.

(3) Each of the imperfection types should be classified separately: the lowest fabrication tolerance quality class obtained corresponding to a high tolerance, should then govern the entire design.

(4) The different tolerance types may each be treated independently, and no interactions need normally be considered.

(5) It should be established by representative sample checks on the completed structure that the measurements of the geometrical imperfections are within the geometrical tolerances stipulated in 8.4.2 to 8.4.5.

(6) Sample imperfection measurements should be undertaken on the unloaded structure (except for self weight) and, where possible, with the operational boundary conditions.

(7) If the measurements of geometrical imperfections do not satisfy the geometrical tolerances stated in 8.4.2 to 8.4.4, any correction steps, such as straightening, should be investigated and decided individually.

> **NOTE:** Before a decision is made in favour of straightening to reduce geometric imperfections, it should be noted that this can cause additional residual stresses. The degree to which the design buckling resistances are utilised in the design should also be considered.

8.4.2 Out-of-roundness tolerance

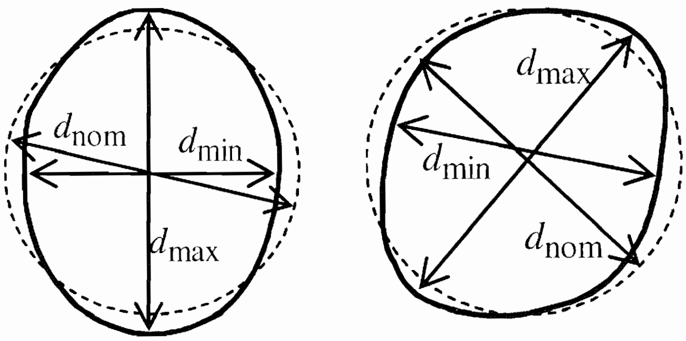
(1) The out-of-roundness should be assessed in terms of the parameter U_r (see figure 8.2) given by:

$$U_r = \frac{d_{max} - d_{min}}{d_{nom}} \quad \text{... (8.1)}$$

where:

d_{max}	is	the maximum measured internal diameter,
d_{min}	is	the minimum measured internal diameter,
d_{nom}	is	the nominal internal diameter.

(2) The measured internal diameter from a given point should be taken as the largest distance across the shell from the point to any other internal point at the same axial coordinate. An appropriate number of diameters should be measured to identify the maximum and minimum values.



a) flattening b) unsymmetrical

Figure 8.2: Measurement of diameters for assessment of out-of-roundness

(3) The out-of-roundness parameter U_r should satisfy the condition:

$$U_r \leq U_{r,max} \quad \text{... (8.2)}$$

where:

$U_{r,max}$ is the out-of-roundness tolerance parameter for the relevant fabrication tolerance quality class.

NOTE: Values for the out-of-roundness tolerance parameter $U_{r,max}$ may be obtained from the National Annex. The recommended values are given in Table 8.1.

Table 8.1: Recommended values for out-of-roundness tolerance parameter $U_{r,max}$

	Diameter range	d [m] ≤ 0,50m	0,50m < d [m] < 1,25m	1,25m ≤ d [m]
Fabrication tolerance quality class	Description	Recommended value of $U_{r,max}$		
Class A	Excellent	0,014	0,007 + 0,0093(1,25–d)	0,007
Class B	High	0,020	0,010 + 0,0133(1,25–d)	0,010
Class C	Normal	0,030	0,015 + 0,0200(1,25–d)	0,015

8.4.3 [AC1⟩ Non-intended ⟨AC1] eccentricity tolerance

(1) At joints in shell walls perpendicular to membrane compressive forces, the [AC1⟩ non-intended ⟨AC1] eccentricity should be evaluated from the measurable total eccentricity e_{tot} and the intended offset e_{int} from:

$$e_a = e_{tot} - e_{int} \qquad \text{... (8.3)}$$

where:

e_{tot} is the eccentricity between the middle surfaces of the joined plates, see figure 8.3c;

e_{int} is the intended offset between the middle surfaces of the joined plates, see figure 8.3b;

e_a is the [AC1⟩ non-intended ⟨AC1] eccentricity between the middle surfaces of the joined plates.

(2) The [AC1⟩ non-intended eccentricity e_a should be less than the maximum permitted non-intended ⟨AC1] eccentricity $e_{a,max}$ for the relevant fabrication tolerance quality class.

NOTE: Values for the maximum permitted [AC1⟩ non-intended ⟨AC1] eccentricity $e_{a,max}$ may be obtained from the National Annex. The recommended values are given in Table 8.2.

Table 8.2: Recommended values for maximum permitted [AC1⟩ non-intended ⟨AC1] eccentricities

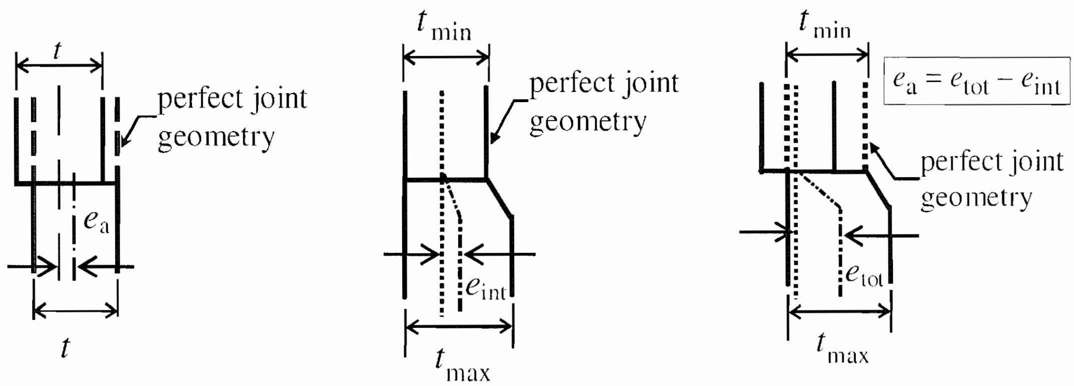
Fabrication tolerance quality class	Description	Recommended values for maximum permitted [AC1⟩ non-intended ⟨AC1] eccentricity $e_{a,max}$
Class A	Excellent	2 mm
Class B	High	3 mm
Class C	Normal	4 mm

(3) The [AC1⟩ non-intended eccentricity e_a should also be assessed in terms of the non-intended ⟨AC1] eccentricity parameter U_e given by:

$$U_e = \frac{e_a}{t_{av}} \text{ or } U_e = \frac{e_a}{t} \qquad \text{... (8.4)}$$

where:

t_{av} is the mean thickness of the thinner and thicker plates at the joint.



a) [AC1⟩ non-intended ⟨AC1] eccentricity when there is no change of plate thickness

b) intended offset at a change of plate thickness without [AC1⟩ non-intended ⟨AC1] eccentricity

c) total eccentricity ([AC1⟩ non-intended ⟨AC1] plus intended) at change of plate thickness

Figure 8.3: [AC1⟩ Non-intended ⟨AC1] eccentricity and intended offset at a joint

(4) The [AC1⟩ non-intended ⟨AC1] eccentricity parameter U_e should satisfy the condition:

$$U_e \leq U_{e,max} \qquad \text{... (8.5)}$$

where:

$U_{e,max}$ is the [AC1⟩ non-intended ⟨AC1] eccentricity tolerance parameter for the relevant fabrication tolerance quality class.

NOTE 1: Values for the [AC1⟩ non-intended ⟨AC1] eccentricity tolerance parameter $U_{e,max}$ may be obtained from the National Annex. The recommended values are given in Table 8.3.

Table 8.3: Recommended values for [AC1⟩ non-intended ⟨AC1] eccentricity tolerances

Fabrication tolerance quality class	Description	Recommended value of $U_{e,max}$
Class A	Excellent	0,14
Class B	High	0,20
Class C	Normal	0,30

NOTE 2: Intended offsets are treated within D.2.1.2 and lapped joints are treated within D.3. These two cases are not treated as imperfections within this standard.

8.4.4 Dimple tolerances

(1) A dimple measurement gauge should be used in every position (see figure 8.4) in both the meridional and circumferential directions. The meridional gauge should be straight, but the gauge for measurements in the circumferential direction should have a curvature equal to the intended radius of curvature r of the middle surface of the shell.

(2) The depth Δw_0 of initial dimples in the shell wall should be measured using gauges of length ℓ_g which should be taken as follows:

a) Wherever meridional compressive stresses are present, including across welds, measurements should be made in both the meridional and circumferential directions, using the gauge of length ℓ_{gx} given by:

$$\ell_{gx} = 4\sqrt{rt} \qquad \text{... (8.6)}$$

b) Where circumferential compressive stresses or shear stresses occur, circumferential direction measurements should be made using the gauge of length $\ell_{g\theta}$ given by:

$$\ell_{g\theta} = 2{,}3\,(\ell^2\,rt)^{0{,}25}, \qquad \text{but} \qquad \ell_{g\theta} \leq r \qquad \text{... (8.7)}$$

where:

ℓ is the meridional length of the shell segment.

c) Additionally, across welds, in both the circumferential and meridional directions, the gauge length ℓ_{gw} should be used:

$$\ell_{gw} = 25\,t \quad \text{or} \quad \ell_{gw} = 25\,t_{min}\,, \quad \text{but with } \ell_{gw} \leq 500\text{mm} \qquad \text{... (8.8)}$$

where:

t_{min} is the thickness of the thinnest plate at the weld.

(3) The depth of initial dimples should be assessed in terms of the dimple parameters U_{0x}, $U_{0\theta}$, U_{0w} given by:

$$U_{0x} = \Delta w_{0x}/\ell_{gx} \qquad U_{0\theta} = \Delta w_{0\theta}/\ell_{g\theta} \qquad U_{0w} = \Delta w_{0w}/\ell_{gw} \qquad \text{... (8.9)}$$

(4) The value of the dimple parameters U_{0x}, $U_{0\theta}$, U_{0w} should satisfy the conditions:

$$U_{0x} \leq U_{0,max} \qquad U_{0\theta} \leq U_{0,max} \qquad U_{0w} \leq U_{0,max} \qquad \text{... (8.10)}$$

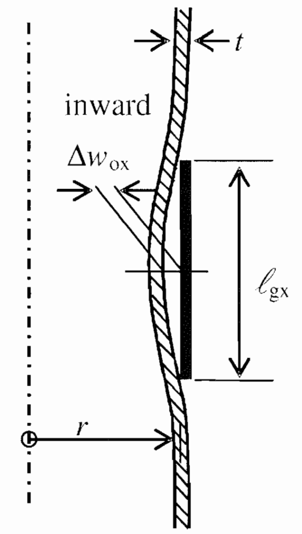
where:

$U_{0,max}$ is the dimple tolerance parameter for the relevant fabrication tolerance quality class.

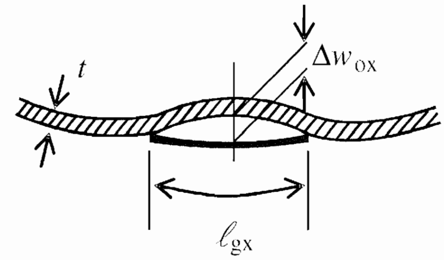
NOTE 1: Values for the dimple tolerance parameter $U_{0,max}$ may be obtained from the National Annex. The recommended values are given in Table 8.4.

Table 8.4: Recommended values for dimple tolerance parameter $U_{0,max}$

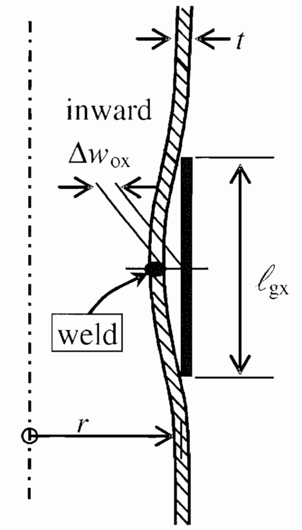
Fabrication tolerance quality class	Description	Recommended value of $U_{0,max}$
Class A	Excellent	0,006
Class B	High	0,010
Class C	Normal	0,016



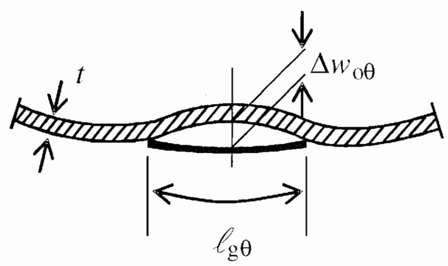
a) Measurement on a meridian (see 8.4.4(2)a)



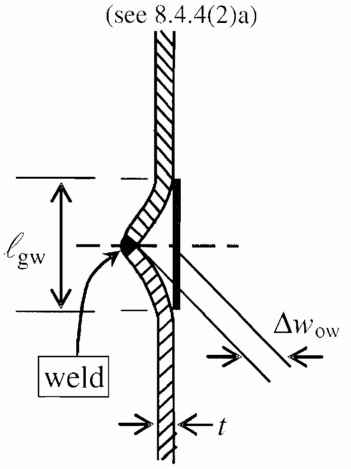
b) First measurement on a circumferential circle (see 8.4.4(2)a)



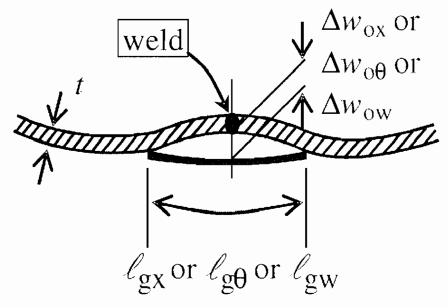
c) First measurement on a meridian across a weld (see 8.4.4(2)a)



d) Second measurement on circumferential circle (see 8.4.4(2)b)



e) Second measurement across a weld with special gauge (see 8.4.4(2)c)



f) Measurements on circumferential circle across weld (see 8.4.4(2)c)

Figure 8.4: Measurement of depths Δw_0 of initial dimples

8.4.5 Interface flatness tolerance

(1) Where another structure continuously supports a shell (such as a foundation), its deviation from flatness at the interface should not include a local slope in the circumferential direction greater than β_θ.

> **NOTE:** The National Annex may choose the value of β_θ. The value $\beta_\theta = 0{,}1\% = 0{,}001$ radians is recommended.

8.5 Stress design

8.5.1 Design values of stresses

(1) The design values of stresses $\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and $\tau_{x\theta,Ed}$ should be taken as the key values of compressive and shear membrane stresses obtained from linear shell analysis (LA). Under purely axisymmetric conditions of loading and support, and in other simple load cases, membrane theory may generally be used.

(2) The key values of membrane stresses should be taken as the maximum value of each stress at that axial coordinate in the structure, unless specific provisions are given in Annex D of this Standard or the relevant application part of EN 1993.

> **NOTE:** In some cases (e.g. stepped walls under circumferential compression, see Annex D.2.3), the key values of membrane stresses are fictitious and larger than the real maximum values.

(3) For basic loading cases the membrane stresses may be taken from Annex A or Annex C.

8.5.2 Design resistance (buckling strength)

(1) The buckling resistance should be represented by the buckling stresses as defined in 1.3.6. The design buckling stresses should be obtained from:

$$\sigma_{x,Rd} = \sigma_{x,Rk}/\gamma_{M1}, \quad \sigma_{\theta,Rd} = \sigma_{\theta,Rk}/\gamma_{M1}, \quad \tau_{x\theta,Rd} = \tau_{x\theta,Rk}/\gamma_{M1} \qquad \text{... (8.11)}$$

(2) The partial factor for resistance to buckling γ_{M1} should be taken from the relevant application standard.

> **NOTE:** The value of the partial factor γ_{M1} may be defined in the National Annex. Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{M1}, it is recommended that the value of γ_{M1} should not be taken as smaller than $\gamma_{M1} = 1{,}1$.

(3) The characteristic buckling stresses should be obtained by multiplying the characteristic yield strength by the buckling reduction factors χ:

$$\sigma_{x,Rk} = \chi_x f_{yk}, \quad \sigma_{\theta,Rk} = \chi_\theta f_{yk}, \quad \tau_{x\theta,Rk} = \chi_\tau f_{yk} / \sqrt{3} \qquad \text{... (8.12)}$$

(4) The buckling reduction factors χ_x, χ_θ and χ_τ should be determined as a function of the relative slenderness of the shell $\overline{\lambda}$ from:

$$\chi = 1 \qquad \text{when} \qquad \overline{\lambda} \le \overline{\lambda}_0 \qquad \text{... (8.13)}$$

$$\chi = 1 - \beta\left(\frac{\overline{\lambda} - \overline{\lambda}_0}{\overline{\lambda}_p - \overline{\lambda}_0}\right)^{\eta} \qquad \text{when} \qquad \overline{\lambda}_0 < \overline{\lambda} < \overline{\lambda}_p \qquad \text{... (8.14)}$$

$$\chi = \frac{\alpha}{\overline{\lambda}^2} \qquad \text{when} \qquad \overline{\lambda}_p \leq \overline{\lambda} \qquad \text{... (8.15)}$$

where:

α	is	the elastic imperfection reduction factor
β	is	the plastic range factor
η	is	the interaction exponent
$\overline{\lambda}_0$	is	the squash limit relative slenderness

NOTE 1: The values of these parameters should be taken from Annex D. Where Annex D does not define the values of these parameters, they may be given by the National Annex.

NOTE 2: Expression (8.15) describes the elastic buckling stress, accounting for geometric imperfections. In this case, where the behaviour is entirely elastic, the characteristic buckling stresses may alternatively be determined directly from $\sigma_{x,Rk} = \alpha_x\ \sigma_{x,Rcr}$, $\sigma_{\theta,Rk} = \alpha_\theta\ \sigma_{\theta,Rcr}$, and $\tau_{x\theta,Rk} = \alpha_\tau\ \tau_{x\theta,Rcr}$.

(5) The value of the plastic limit relative slenderness $\overline{\lambda}_p$ should be determined from:

$$\overline{\lambda}_p = \sqrt{\frac{\alpha}{1-\beta}} \qquad \text{... (8.16)}$$

(6) The relative shell slenderness parameters for different stress components should be determined from:

$$\overline{\lambda}_x = \sqrt{f_{yk} / \sigma_{x,Rcr}}\ , \qquad \overline{\lambda}_\theta = \sqrt{f_{yk} / \sigma_{\theta,Rcr}}\ , \qquad \overline{\lambda}_\tau = \sqrt{\left(f_{yk} / \sqrt{3}\right) / \tau_{x\theta,Rcr}} \qquad \text{... (8.17)}$$

(7) The elastic critical buckling stresses $\sigma_{x,Rcr}$, $\sigma_{\theta,Rcr}$ and $\tau_{x\theta,Rcr}$ should be obtained by means of the relevant expressions in Annex D.

(8) Where no appropriate expressions are given in Annex D, the elastic critical buckling stresses may be extracted from a numerical LBA analysis of the shell under the buckling-relevant combinations of actions defined in 8.1. For the conditions that this analysis must satisfy, see 8.6.2 (5) and (6).

8.5.3 Stress limitation (buckling strength verification)

(1) Although buckling is not a purely stress-initiated failure phenomenon, the buckling limit state, within this section, should be represented by limiting the design values of membrane stresses. The influence of bending effects on the buckling strength may be neglected provided they arise as a result of meeting boundary compatibility requirements. In the case of bending stresses from local loads or from thermal gradients, special consideration should be given.

(2) Depending on the loading and stressing situation, one or more of the following checks for the key values of single membrane stress components should be carried out:

$$\sigma_{x,Ed} \leq \sigma_{x,Rd}, \qquad \sigma_{\theta,Ed} \leq \sigma_{\theta,Rd}, \qquad \tau_{x\theta,Ed} \leq \tau_{x\theta,Rd} \qquad \text{... (8.18)}$$

(3) If more than one of the three buckling-relevant membrane stress components are present under the actions under consideration, the following interaction check for the combined membrane stress state should be carried out:

$$\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)^{k_x} - k_i \left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)\left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right) + \left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right)^{k_\theta} + \left(\frac{\tau_{x\theta,Ed}}{\tau_{x\theta,Rd}}\right)^{k_\tau} \leq 1 \qquad \text{... (8.19)}$$

where $\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and $\tau_{x\theta,Ed}$ are the interaction-relevant groups of the significant values of compressive and shear membrane stresses in the shell and the values of the buckling interaction parameters k_x, k_θ , k_τ and k_i are given in Annex D.

(4) Where $\sigma_{x,Ed}$ or $\sigma_{\theta,Ed}$ is tensile, its value should be taken as zero in expression (8.19).

NOTE: For axially compressed cylinders with internal pressure (leading to circumferential tension) special provisions are made in Annex D. The resulting value of $\sigma_{x,Rd}$ accounts for both the strengthening effect of internal pressure on the elastic buckling resistance and the weakening effect of the elastic-plastic elephant's foot phenomenon (expression D.43). If the tensile $\sigma_{\theta,Ed}$ is then taken as zero in expression (8.19), the buckling strength is accurately represented.

(5) The locations and values of each of the buckling-relevant membrane stresses to be used together in combination in expression (8.19) are defined in Annex D.

(6) Where the shell buckling condition is not included in Annex D, the buckling interaction parameters may be conservatively estimated using:

$$k_x = 1{,}0 + \chi_x^2 \qquad \text{...(8.20)}$$

$$k_\theta = 1{,}0 + \chi_\theta^2 \qquad \text{...(8.21)}$$

$$k_\tau = 1{,}5 + 0{,}5\,\chi_\tau^2 \qquad \text{...(8.22)}$$

$$k_i = (\chi_x\,\chi_\theta)^2 \qquad \text{...(8.23)}$$

NOTE: These rules may sometimes be very conservative, but they include the two limiting cases which are well established as safe for a wide range of cases:
a) in very thin shells, the interaction between σ_x and σ_θ is approximately linear; and
b) in very thick shells, the interaction becomes that of von Mises.

8.6 Design by global numerical analysis using MNA and LBA analyses

8.6.1 Design value of actions

(1) The design values of actions should be taken as in 8.1 (1).

8.6.2 Design value of resistance

(1) The design buckling resistance should be determined from the amplification factor r_{Rd} applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The design buckling resistance $F_{Rd} = r_{Rd} \cdot F_{Ed}$ should be obtained from the plastic reference resistance $F_{R_{pl}} = r_{Rpl} \cdot F_{Ed}$ and the elastic critical buckling resistance $F_{cr} = r_{Rcr} \cdot F_{Ed}$, combining these to find the characteristic buckling resistance $F_{R_k} = r_{Rk} \cdot F_{Ed}$. The partial factor γ_{M1} should then be used to obtain the design resistance.

(3) The plastic reference resistance ratio r_{Rpl} (see figure 8.5) should be obtained by materially non-linear analysis (MNA) as the plastic limit load under the applied combination of actions. This load ratio r_{Rpl} may be taken as the largest value attained in the analysis, ignoring the effect of strain hardening.

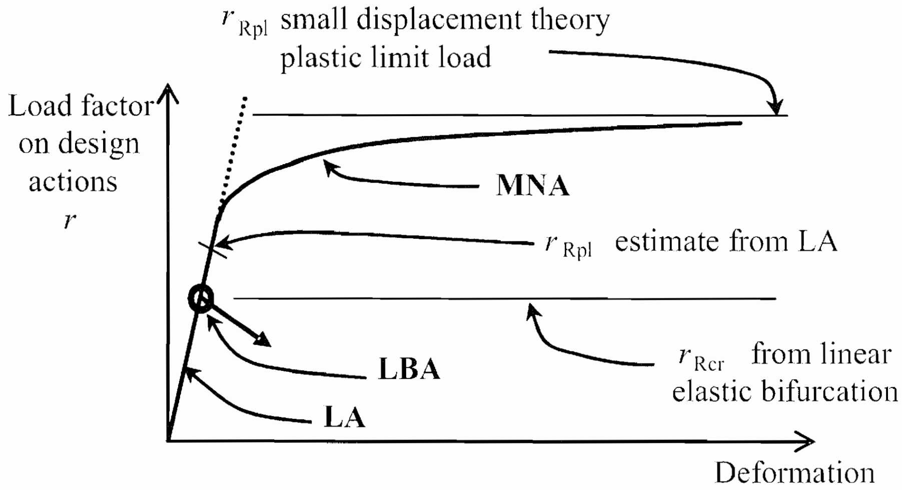


Figure 8.5: Definition of plastic reference resistance ratio r_{Rpl} and critical buckling resistance ratio r_{Rcr} derived from global MNA and LBA analyses

(4) Where it is not possible to undertake a materially non-linear analysis (MNA), the plastic reference resistance ratio r_{Rpl} may be conservatively estimated from linear shell analysis (LA) conducted using the design values of the applied combination of actions using the following procedure. The evaluated membrane stress resultants $n_{\text{x,Ed}}$, $n_{\theta\text{,Ed}}$ and $n_{\text{x}\theta\text{,Ed}}$ at any point in the shell should be used to estimate the plastic reference resistance from:

$$r_{Rpl} = \frac{t \cdot f_{yk}}{\sqrt{n_{x,Ed}^2 - n_{x,Ed} \cdot n_{\theta,Ed} + n_{\theta,Ed}^2 + n_{x\theta,Ed}^2}} \quad \text{... (8.24)}$$

The lowest value of plastic resistance ratio so calculated should be taken as the estimate of the plastic reference resistance ratio r_{Rpl}.

> **NOTE:** A safe estimate of r_{Rpl} can usually be obtained by applying expression (8.24) in turn at the three points in the shell where each of the three buckling-relevant membrane stress resultants attains its highest value, and using the lowest of these three estimates as the relevant value of r_{Rpl}.

(5) The elastic critical buckling resistance ratio r_{Rcr} should be determined from an eigenvalue analysis (LBA) applied to the linear elastic calculated stress state in the geometrically perfect shell (LA) under the design values of the load combination. The lowest eigenvalue (bifurcation load factor) should be taken as the elastic critical buckling resistance ratio r_{Rcr}, see figure 8.5.

(6) It should be verified that the eigenvalue algorithm that is used is reliable at finding the eigenmode that leads to the lowest eigenvalue. In cases of doubt, neighbouring eigenvalues and their eigenmodes should be calculated to obtain a fuller insight into the bifurcation behaviour of the shell. The analysis should be carried out using software that has been authenticated against benchmark cases with physically similar buckling characteristics.

(7) The overall relative slenderness λ_{ov} for the complete shell should be determined from:

$$\overline{\lambda}_{\text{ov}} = \sqrt{F_{\text{Rpl}} / F_{\text{Rcr}}} = \sqrt{r_{\text{Rpl}} / r_{\text{Rcr}}} \quad \text{... (8.25)}$$

(8) The overall buckling reduction factor χ_{ov} should be determined as

$\chi_{\text{ov}} = f\left(\overline{\lambda}_{\text{ov}}, \overline{\lambda}_{\text{ov,0}}, \alpha_{\text{ov}}, \beta_{\text{ov}}, \eta_{\text{ov}}\right)$ using 8.5.2 (4), in which α_{ov} is the overall elastic imperfection reduction factor, β_{ov} is the plastic range factor, η_{ov} is the interaction exponent and $\overline{\lambda}_{ov,0}$ is the squash limit relative slenderness.

(9) The evaluation of the factors $\overline{\lambda}_{ov,0}$, r_{Rov}, β_{ov} and η_{ov} should take account of the imperfection sensitivity, geometric nonlinearity and other aspects of the particular shell buckling case. Conservative values for these parameters should be determined by comparison with known shell buckling cases (see Annex D) that have similar buckling modes, similar imperfection sensitivity, similar geometric nonlinearity, similar yielding sensitivity and similar postbuckling behaviour. The value of r_{Rov} should also take account of the appropriate fabrication tolerance quality class.

> **NOTE:** Care should be taken in choosing an appropriate value of r_{Rov} when this approach is used on shell geometries and loading cases where snap-through buckling may occur. Such cases include conical and spherical caps and domes under external pressure or on supports that can displace radially. The appropriate value of r_{Rov} should also be chosen with care when the shell geometry and load case produce conditions that are highly sensitive to changes of geometry, such as at unstiffened junctions between cylindrical and conical shell segments under meridional compressive loads (e.g. in chimneys).
>
> The commonly reported elastic shell buckling loads for these special cases are normally based on geometrically nonlinear analysis applied to a perfect or imperfect geometry, which predicts the snap-through buckling load. By contrast, the methodology used here adopts the linear bifurcation load as the reference elastic critical buckling resistance, and this is often much higher than the snap-through load. The design calculation must account for these two sources of reduced resistance by an appropriate choice of the overall elastic imperfection reduction factor r_{Rov}. This choice must include the effect of both the geometric nonlinearity (that can lead to snap-through) and the additional strength reduction caused by geometric imperfections.

(10) If the provisions of (9) cannot be achieved beyond reasonable doubt, appropriate tests should be carried out, see EN1990, Annex D.

(11) If specific values of r_{Rov}, β_{ov}, η_{ov} and $\overline{\lambda}_{ov,0}$ are not available according to (9) or (10), the values for an axially compressed unstiffened cylinder may be adopted, see D.1.2.2. Where snap-through is known to be a possibility, appropriate further reductions in r_{Rov} should be considered.

(12) The characteristic buckling resistance ratio r_{Rk} should be obtained from:

$$r_{Rk} = \chi_{ov}\, r_{Rpl} \qquad \text{... (8.26)}$$

where:

r_{Rpl} is the plastic reference resistance ratio.

(13) The design buckling resistance ratio r_{Rd} should be obtained from:

$$r_{Rd} = r_{Rk}/\gamma_{M1} \qquad \text{... (8.27)}$$

where:

γ_{M1} is the partial factor for resistance to buckling according to 8.5.2 (2).

8.6.3 Buckling strength verification

(1) It should be verified that:

$$F_{Ed} \leq F_{Rd} = r_{Rd} \cdot F_{Ed} \quad \text{or} \quad r_{Rd} \geq 1 \qquad \text{... (8.28)}$$

8.7 Design by global numerical analysis using GMNIA analysis

8.7.1 Design values of actions

(1) The design values of actions should be taken as in 8.1 (1).

8.7.2 Design value of resistance

(1) The design buckling resistance should be determined as a load factor r_R applied to the design values F_{Ed} of the combination of actions for the relevant load case.

(2) The characteristic buckling resistance ratio r_{Rk} should be found from the imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$, adjusted by the calibration factor k_{GMNIA}. The design buckling resistance ratio r_{Rd} should then be found using the partial factor γ_{M1}.

(3) To determine the imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$, a GMNIA analysis of the geometrically imperfect shell under the applied combination of actions should be carried out, accompanied by an eigenvalue analysis to detect possible bifurcations in the load path.

> **NOTE:** Where plasticity has a significant effect on the buckling resistance, care should be taken to ensure that the adopted imperfection mode induces some pre-buckling shear strains, because the shear modulus is very sensitive to small plastic shear strains. In certain shell buckling problems (e.g. shear buckling of annular plates), if this effect is omitted, the eigenvalue analysis may give a considerable overestimate of the elastic-plastic buckling resistance.

(4) An LBA analysis should first be performed on the perfect structure to determine the elastic critical buckling resistance ratio r_{Rcr} of the perfect shell. An MNA should next be performed on the perfect structure to determine the perfect plastic reference resistance ratio r_{Rpl}. These two resistance ratios should then be used to establish the overall relative slenderness $\bar{\lambda}_{ov}$ for the complete shell according to expression 8.25.

(5) A GMNA analysis should then be performed on the perfect structure to determine the perfect elastic-plastic buckling resistance ratio $r_{R,GMNA}$. This resistance ratio should be used later to verify that the effect of the chosen geometric imperfections has a sufficiently deleterious effect to give confidence that the lowest resistance has been obtained. The GMNA analysis should be carried out under the applied combination of actions, accompanied by an eigenvalue analysis to detect possible bifurcations in the load path.

(6) The imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$ should be found as the lowest load factor r_R obtained from the three following criteria C1, C2 and C3, see figure 8.6:

Criterion C1: The maximum load factor on the load-deformation-curve (limit load);
Criterion C2: The bifurcation load factor, where this occurs during the loading path before reaching the limit point of the load-deformation-curve;
Criterion C3: The largest tolerable deformation, where this occurs during the loading path before reaching a bifurcation load or a limit load.

(7) The largest tolerable deformation should be assessed relative to the conditions of the individual structure. If no other value is available, the largest tolerable deformation may be deemed to have been reached when the greatest local rotation of the shell surface (slope of the surface relative to its original geometry) attains the value β.

> **NOTE:** The National Annex may choose the value of β. The value β = 0,1 radians is recommended.

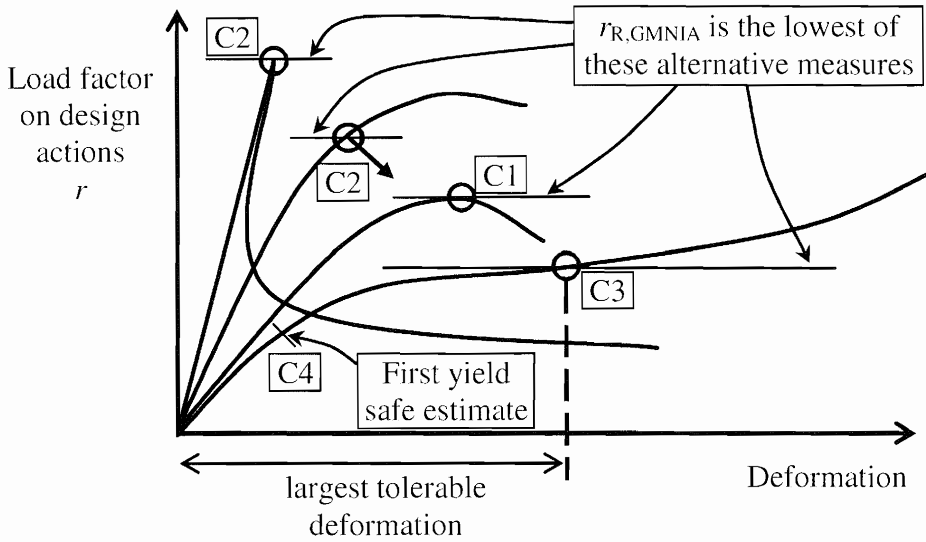


Figure 8.6: Definition of buckling resistance from global GMNIA analysis

(8) A conservative assessment of the imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$ may be obtained using a GNIA analysis of the geometrically imperfect shell under the applied combination of actions. In this case, the following criterion should be used to determine the lowest load factor r_R:

Criterion C4: The load factor at which the equivalent stress at the most highly stressed point on the shell surface reaches the design value of the yield stress $f_{yd} = f_{yk}/\gamma_{M0}$, see figure 8.6.

NOTE: It should be noted that GMNA, GMNIA and GNIA analyses must always be undertaken with regular eigenvalue checks to ensure that any possible bifurcation on the load path is detected.

(9) In formulating the GMNIA (or GNIA) analysis, appropriate allowances should be incorporated to cover the effects of imperfections that cannot be avoided in practice, including:

a) geometric imperfections, such as:
deviations from the nominal geometric shape of the middle surface (pre-deformations, out-of-roundness);
irregularities at and near welds (minor eccentricities, shrinkage depressions, rolling curvature errors);
deviations from nominal thickness;
lack of evenness of supports.

b) material imperfections, such as:
residual stresses caused by rolling, pressing, welding, straightening etc.;
inhomogeneities and anisotropies.

NOTE: Further possible negative influences on the imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$, such as ground settlements or flexibilities of connections or supports, are not classed as imperfections in the sense of these provisions.

(10) Imperfections should be allowed for in the GMNIA analysis by including appropriate additional quantities in the analytical model for the numerical computation.

(11) The imperfections should generally be introduced by means of equivalent geometric imperfections in the form of initial shape deviations perpendicular to the middle surface of the perfect shell, unless a better technique is used. The middle surface of the geometrically imperfect shell should be obtained by superposition of the equivalent geometric imperfections on the perfect shell geometry.

(12) The pattern of the equivalent geometric imperfections should be chosen in such a form that it has the most unfavourable effect on the imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$ of the shell. If the most unfavourable pattern cannot be readily identified beyond reasonable doubt, the analysis should be carried out for a sufficient number of different imperfection patterns, and the worst case (lowest value of $r_{R,GMNIA}$) should be identified.

(13) The eigenmode-affine pattern should be used unless a different unfavourable pattern can be justified.

NOTE: The eigenmode affine pattern is the critical buckling mode associated with the elastic critical buckling resistance ratio r_{Rcr} based on an LBA analysis of the perfect shell.

(14) The pattern of the equivalent geometric imperfections should, if practicable, reflect the constructional detailing and the boundary conditions in an unfavourable manner.

(15) Notwithstanding (13) and (14), patterns may be excluded from the investigation if they can be eliminated as unrealistic because of the method of fabrication, manufacture or erection.

(16) Modification of the adopted mode of geometric imperfections to include realistic structural details (such as axisymmetric weld depressions) should be explored.

NOTE: The National Annex may define additional requirements for the assessment of appropriate patterns of imperfections.

(17) The sign of the equivalent geometric imperfections should be chosen in such a manner that the maximum initial shape deviations are unfavourably oriented towards the centre of the shell curvature.

(18) The amplitude of the adopted equivalent geometric imperfection form should be taken as dependent on the fabrication tolerance quality class. The maximum deviation of the geometry of the equivalent imperfection from the perfect shape $\Delta w_{0,eq}$ should be the larger of $\Delta w_{0,eq,1}$ and $\Delta w_{0,eq,2}$, where:

$$\Delta w_{0,eq,1} = \ell_g \; U_{n1} \qquad \text{... (8.29)}$$

$$\Delta w_{0,eq,2} = n_i \; t \; U_{n2} \qquad \text{... (8.30)}$$

where:

ℓ_g is all relevant gauge lengths according to 8.4.4 (2);
t is the local shell wall thickness;
n_i is a multiplier to achieve an appropriate tolerance level;
U_{n1} and U_{n2} are the dimple imperfection amplitude parameters for the relevant fabrication tolerance quality class.

NOTE 1: The National Annex may choose the value of n_i. The value $n_i = 25$ is recommended.

NOTE 2: Values for the dimple tolerance parameter U_{n1} and U_{n2} may be obtained from the National Annex. The recommended values are given in Table 8.5.

Table 8.5: Recommended values for dimple imperfection amplitude parameters U_{n1} and U_{n2}

Fabrication tolerance quality class	Description	Recommended value of U_{n1}	Recommended value of U_{n2}
Class A	Excellent	0,010	0,010
Class B	High	0,016	0,016
Class C	Normal	0,025	0,025

(19) The amplitude of the geometric imperfection in the adopted pattern of the equivalent geometric imperfection should be interpreted in a manner which is consistent with the gauge length method, set out in 8.4.4 (2), by which it is defined.

(20) Additionally, it should be verified that an analysis that adopts an imperfection whose amplitude is 10% smaller than the value $\Delta w_{0,eq}$ found in (18) does not yield a lower value for the ratio $r_{R,GMNIA}$. If a lower value is obtained, the procedure should be iterated to find the lowest value of the ratio $r_{R,GMNIA}$ as the amplitude is varied.

(21) If follower load effects are possible, either they should be incorporated in the analysis, or it should be verified that their influence is negligible.

(22) For each calculated value of the imperfect elastic-plastic buckling resistance $r_{R,GMNIA}$, the ratio of the imperfect to perfect resistance ($r_{R,GMNIA}/r_{R,GMNA}$) should be determined and compared with values of r_R found using the procedures of 8.5 and Annex D, to verify that the chosen geometric imperfection has a deleterious effect that is comparable with that obtained from a lower bound to test results.

NOTE: Where the resistance is dominated by plasticity effects, the ratio ($r_{R,GMNIA}/r_{R,GMNA}$) will be much larger than the elastic imperfection reduction factor α, and no close comparison can be expected. However, where the resistance is controlled by buckling phenomena that are substantially elastic, the ratio ($r_{R,GMNIA}/r_{R,GMNA}$) should be only a little higher than the value determined by hand calculation, and the factors leading to the higher value should be considered.

(23) The reliability of the numerically determined imperfect elastic-plastic buckling resistance ratio $r_{R,GMNIA}$ should be checked by one of the following alternative methods:

a) by using the same program to calculate values $r_{R,GMNIA,check}$ for other shell buckling cases for which characteristic buckling resistance ratio values $r_{Rk,known,check}$ are known. The check cases should use basically similar imperfection assumptions and be similar in their buckling controlling parameters (such as relative shell slenderness, postbuckling behaviour, imperfection-sensitivity, geometric nonlinearity and material behaviour);

b) by comparison of calculated values ($r_{R,GMNIA,check}$) against test results ($r_{R,test,known,check}$). The check cases should satisfy the same similarity conditions given in (a).

NOTE 1: Other shell buckling cases for which the characteristic buckling resistance ratio values $r_{Rk,known,check}$ are known may be found from the scientific literature on shell buckling. It should be noted that the hand calculations of 8.5 and Annex D are derived as general lower bounds on test results, and these sometimes lead to such low assessed values for the characteristic buckling resistance that they cannot be easily obtained numerically.

NOTE 2: Where test results are used, it should be established that the geometric imperfections present in the test may be expected to be representative of those that will occur in practical construction.

(24) Depending on the results of the reliability checks, the calibration factor k_{GMNIA} should be evaluated, as appropriate, from:

$$k_{GMNIA} = \frac{r_{Rk,known,check}}{r_{R,GMNIA,check}} \quad \text{or} \quad k_{GMNIA} = \frac{r_{R,test,known,check}}{r_{R,GMNIA,check}} \qquad \text{... (8.31)}$$

where:

$r_{Rk,known,check}$ is the known characteristic value;

$r_{R,test,known,check}$ is the known test result;

$r_{R,GMNIA,check}$ is the calculation outcome for the check buckling case or the test buckling case, as appropriate.

(25) Where test results are used to determine k_{GMNIA}, and the calculated value of k_{GMNIA} exceeds 1,0, the adopted value should be $k_{GMNIA} = 1{,}0$.

(26) Where a known characteristic value based on existing established theory is used to determine k_{GMNIA}, and the calculated value of k_{GMNIA} lies outside the range $0{,}8 < k_{GMNIA} < 1{,}2$, this procedure should not be used. The GMNIA result should be deemed invalid and further calculations undertaken to establish the causes of the discrepancy.

(27) The characteristic buckling resistance ratio should be obtained from:

$$r_{Rk} = k_{GMNIA} \; r_{R,GMNIA} \qquad \text{... (8.32)}$$

where:

$r_{R,GMNIA}$ is the calculated imperfect elastic-plastic buckling resistance ratio;
k_{GMNIA} is the calibration factor.

8.7.3 Buckling strength verification

(1) The design buckling resistance ratio r_{Rd} should be obtained from:

$$r_{Rd} = r_{Rk} / \gamma_{M1} \qquad \text{... (8.33)}$$

where:

γ_{M1} is the partial factor for resistance to buckling according to 8.5.2 (2).

(2) It should be verified that:

$$F_{Ed} \le F_{Rd} = r_{Rd} \cdot F_{Ed} \quad \text{or} \quad r_{Rd} \ge 1 \qquad \text{... (8.34)}$$

9 Fatigue limit state (LS4)

9.1 Design values of actions

(1) The design values of the actions for each load case should be taken as the varying parts of the total action representing the anticipated action spectrum throughout the design life of the structure.

(2) The relevant action spectra should be obtained from EN 1991 in accordance with the definitions given in the appropriate application parts of EN 1993.

9.2 Stress design

9.2.1 General

(1) The fatigue assessment presented in EN 1993-1-9 should be used, except as provided here.

(2)P The partial factor for resistance to fatigue γ_{Mf} shall be taken from the relevant application standard.

> **NOTE:** The value of the partial factor γ_{Mf} may be defined in the National Annex. Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{Mf}, the value of γ_{Mf} should be taken from EN 1993-1-9. It is recommended that the value of γ_{Mf} should not be taken as smaller than $\gamma_{Mf} = 1,1$.

9.2.2 Design values of stress range

(1) Stresses should be determined by a linear elastic analysis of the structure subject to the design values of the fatigue actions.

(2) In each verification of the limit state, the design value of the fatigue stress should be taken as the larger stress range $\Delta\sigma$ of the values on the two surfaces of the shell, and based on the sum of the primary and the secondary stresses.

(3) Depending upon the fatigue assessment carried out according to EN 1993-1-9, either nominal stress ranges or geometric stress ranges should be evaluated.

(4) Nominal stress ranges may be used if 9.2.3 (2) is adopted.

(5) Geometric stress ranges should be used for construction details that differ from those of 9.2.3 (2).

(6) The geometric stress range takes into account only the overall geometry of the joint, excluding local stresses due to the weld geometry and internal weld effects. It may be determined by use of geometrical stress concentration factors given by expressions.

(7) Stresses used for the fatigue design of construction details with linear geometric orientation should be resolved into components transverse to and parallel to the axis of the detail.

9.2.3 Design values of resistance (fatigue strength)

(1) The design values of resistance obtained from the following may be applied to structural steels in the temperature range up to 150° C.

(2) The fatigue resistance of construction details commonly found in shell structures should be obtained from EN 1993-3-2 in classes and evaluated in terms of the stress range $\Delta\sigma_E$, appropriate to the number of cycles, in which the values are additionally classified according to the quality of the welds.

(3) The fatigue resistance of the detail classes should be obtained from EN 1993-1-9.

9.2.4 Stress range limitation

(1) In every verification of this limit state, the design stress range $\Delta\sigma_E$ should satisfy the condition:

$$\gamma_{Ff}\ \Delta\sigma_E \leq \Delta\sigma_R / \gamma_{Mf} \qquad \text{... (9.1)}$$

where:

γ_{Ff}	is	the partial factor for the fatigue loading
γ_{Mf}	is	the partial factor for the fatigue resistance
$\Delta\sigma_E$	is	the equivalent constant amplitude stress range of the design stress spectrum
$\Delta\sigma_R$	is	the fatigue strength stress range for the relevant detail category and the number of cycles of the stress spectrum

(2) As an alternative to (1), a cumulative damage assessment may be made for a set of m different stress ranges $\Delta\sigma_i$ (i = 1,m) using the Palmgren-Miner rule:

$$D_d \leq 1 \qquad \text{... (9.2)}$$

in which:

$$D_d = \sum_{i=1}^{m} n_i / N_i \qquad \text{... (9.3)}$$

where:

n_i	is	the number of cycles of the stress range $\Delta\sigma_i$
N_i	is	the number of cycles of the stress range $\gamma_{Ff}\ \gamma_{Mf}\ \Delta\sigma_i$ to cause failure for the relevant detail category

(3) In the case of combination of normal and shear stress ranges the combined effects should be considered in accordance with EN 1993-1-9.

9.3 Design by global numerical LA or GNA analysis

(1) The fatigue design on the basis of an elastic analysis (LA or GNA analysis) should follow the provisions given in 9.2 for stress design. However, the stress ranges due to the fatigue loading should be determined by means of a shell bending analysis, including the geometric discontinuities of joints in constructional details.

(2) If a three dimensional finite element analysis is used, the notch effects due to the local weld geometry should be eliminated.

ANNEX A (normative)
Membrane theory stresses in shells

A.1 General

A.1.1 Action effects and resistances

The action effects or resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the action effect or resistance when characteristic values of the actions, geometric parameters and material properties are adopted.

A.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

F_x	axial load applied to the cylinder
F_z	axial load applied to a cone
M	global bending moment applied to the complete cylinder (not to be confused with the moment per unit width in the shell wall m)
M_t	global torque applied to the complete cylinder
V	global transverse shear applied to the complete cylinder

Roman lower case letters

g	unit weight of the material of the shell
p_n	distributed normal pressure
p_x	distributed axial traction on cylinder wall

Greek lower case letters

ϕ	meridional slope angle
σ_x	axial or meridional membrane stress ($= n_x/t$)
σ_θ	circumferential membrane stress ($= n_\theta/t$)
τ	membrane shear stress ($= n_{x\theta}/t$)

A.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 2.3 and 5.2.2.

(2) For these expressions to be strictly valid, the boundary conditions for cylinders should be taken as radially free at both ends, axially supported at one end, and rotationally free at both ends.

(3) For these expressions to be strictly valid for cones, the applied loading should match a membrane stress state in the shell and the boundary conditions should be taken as free to displace normal to the shell at both ends and meridionally supported at one end.

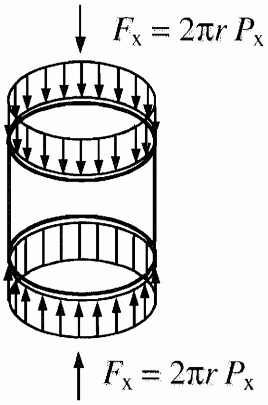
(4) For truncated cones, the boundary conditions should be understood to include components of loading transverse to the shell wall, so that the combined stress resultant introduced into the shell is solely in the direction of the shell meridian.

A.1.4 Sign convention

(1) The sign convention for stresses σ should be taken everywhere as tension positive, though some of the figures illustrate cases in which the external load is applied in the opposite sense.

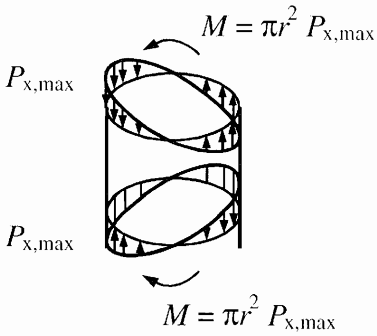
A.2 Unstiffened cylindrical shells

A.2.1 Uniform axial load



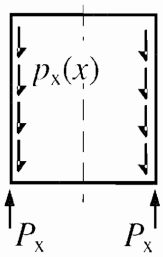
$$\sigma_x = -\frac{F_x}{2\pi r t}$$

A.2.2 Axial load from global bending



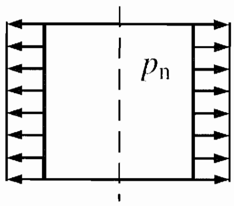
$$\sigma_x = \pm\frac{M}{\pi r^2 t}$$

A.2.3 Friction load



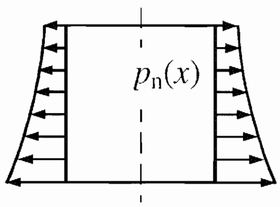
$$\sigma_x = -\frac{1}{t}\int_0^l p_x \cdot dx$$

A.2.4 Uniform internal pressure



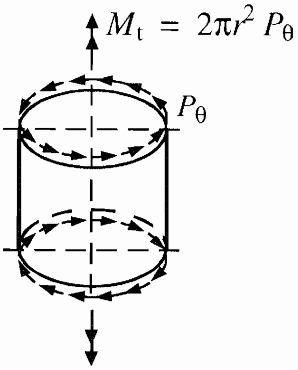
$$\sigma_\theta = p_n \cdot \frac{r}{t}$$

A.2.5 Variable internal pressure



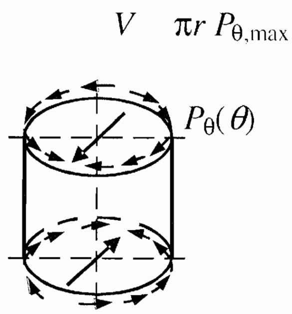
$$\sigma_\theta(x) = p_n(x) \cdot \frac{r}{t}$$

A.2.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

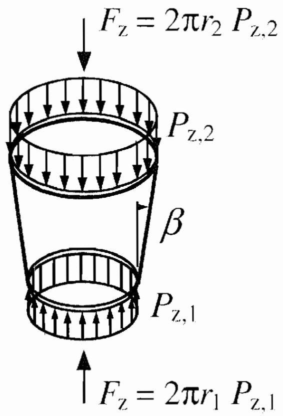
A.2.7 Sinusoidal shear from transverse force



$$\tau_{max} = \pm\frac{V}{\pi r t}$$

A.3 Unstiffened conical shells

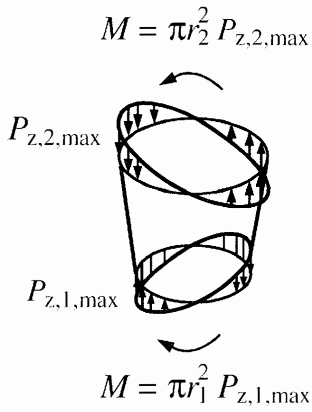
A.3.1 Uniform axial load



$$\sigma_x = -\frac{F_z}{2\pi r t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

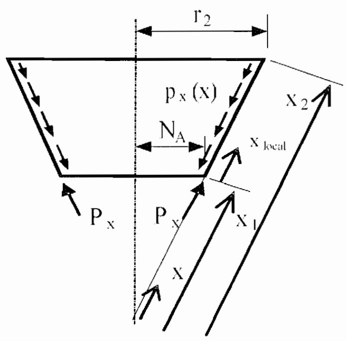
A.3.2 Axial load from global bending



$$\sigma_{x,\max} = \pm\frac{M}{\pi r^2 t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

A.3.3 Friction load

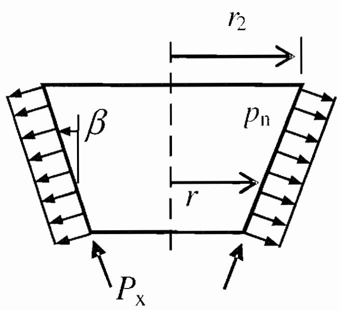


$$x_1 = \frac{r_1}{\sin\beta} \qquad x_2 = \frac{r_2}{\sin\beta}$$

$$\sigma_{x1} = -\frac{1}{x_1 t}\int_{x_1}^{x_2} p_x x \cdot dx$$

$$\sigma_\theta = 0$$

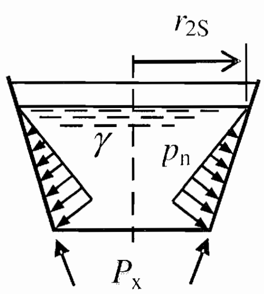
A.3.4 Uniform internal Pressure



$$\sigma_x = -p_n \frac{r}{2t \cdot \cos\beta}\left[\left(\frac{r_2}{r}\right)^2\right]$$

$$\sigma_\theta = p_n \frac{r}{t \cdot \cos\beta}$$

A.3.5 Linearly varying internal pressure

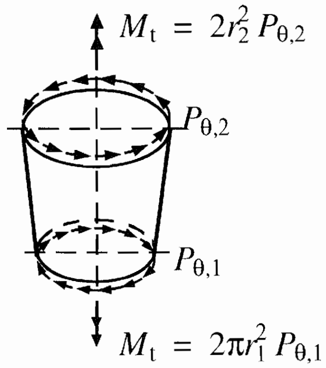


r_{2S} is the radius at the fluid surface

$$\sigma_x = -\frac{\gamma r}{t \cdot \sin\beta}\left\{\frac{r_{2s}}{6}\left[\left(\frac{r_{2s}}{r}\right)^2 - 3\right] + \frac{r}{3}\right\}$$

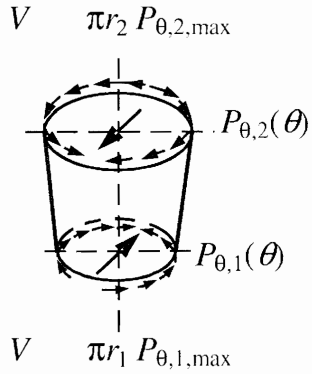
$$\sigma_\theta = +\frac{\gamma r}{t \cdot \sin\beta}(r_{2s} - r)$$

A.3.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

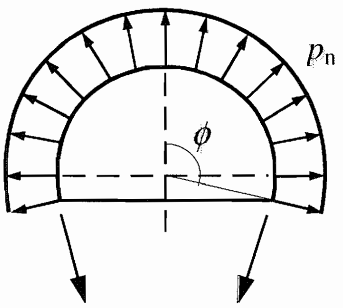
A.3.7 Sinusoidal shear from transverse force



$$\tau_{\max} = \pm \frac{V}{\pi r t}$$

A.4 Unstiffened spherical shells

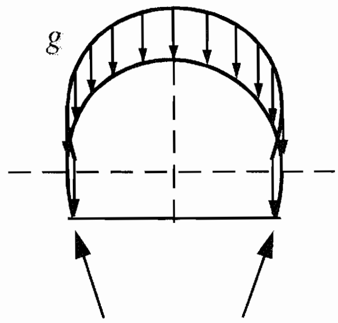
A.4.1 Uniform internal pressure



$$\sigma_x = \frac{p_n r}{2t}$$

$$\sigma_\theta = \frac{p_n r}{2t}$$

A.4.2 Uniform self-weight load



$$\sigma_x = -\frac{gr}{t}\left(\frac{1}{1+\cos\phi}\right)$$

$$\sigma_\theta = -\frac{gr}{t}\left(\cos\phi - \frac{1}{1+\cos\phi}\right)$$

ANNEX B (normative)
Additional expressions for plastic collapse resistances

B.1 General

B.1.1 Resistances

The resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the resistance when characteristic values of the geometric parameters and material properties are adopted.

B.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

A_r	cross-sectional area of a ring
P_R	characteristic value of small deflection theory plastic mechanism resistance

Roman lower case letters

b	thickness of a ring
ℓ	effective length of shell which acts with a ring
r	radius of the cylinder
s_e	dimensionless von Mises equivalent stress parameter
s_m	dimensionless combined stress parameter
s_x	dimensionless axial stress parameter
s_θ	dimensionless circumferential stress parameter

Subscripts

r	relating to a ring
R	resistance

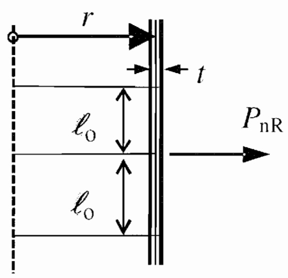
B.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 5.2.2.

(2) The term "clamped" should be taken to refer to BC1r and the term "pinned" to refer to BC2f.

B.2 Unstiffened cylindrical shells

B.2.1 Cylinder: Radial line load



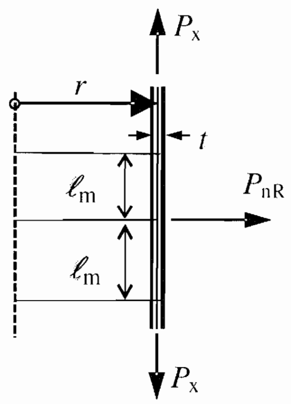
Reference quantities:

$$\ell_o = 0{,}975\sqrt{rt}$$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_0} = f_y \frac{t}{r}$$

B.2.2 Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad \ell_o = 0{,}975\sqrt{rt}$$

Range of applicability:

$$-1 \le s_x \le +1$$

Dependent parameters:

If $P_n > 0$ (outward) then: $A = +s_x - 1{,}50$

If $P_n < 0$ (inward) then: $A = -s_x - 1{,}50$

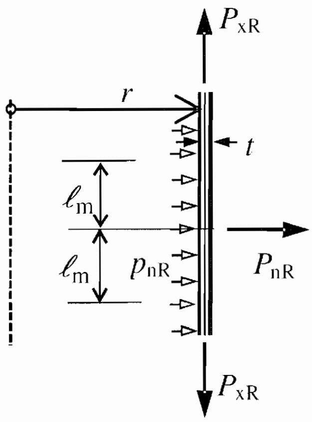
$$s_m = A + \sqrt{A^2 + 4(1 - s_x^2)}$$

If $s_x \neq 0$ then: $\ell_m = s_m\, \ell_o$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_m} = f_y \frac{t}{r}$$

B.2.3 Cylinder: Radial line load, constant internal pressure and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad s_\theta = \frac{p_n}{f_y} \cdot \frac{r}{t}$$

$$\ell_o = 0{,}975\sqrt{rt} \qquad s_e = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$$

Range of applicability:

$$-1 \le s_x \le +1 \qquad -1 \le s_\theta \le +1$$

Dependent parameters:

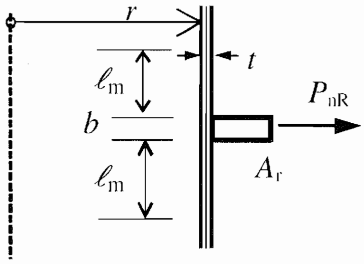
Outward directed ring load $P_n > 0$		Inward directed ring load $P_n < 0$	
Condition	Expressions	Condition	Expressions
$s_e < 1{,}00$ and $s_\theta \le 0{,}975$	$A = +s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 - s_\theta} \right)$	$s_e < 1{,}00$ and $s_\theta \ge -0{,}975$	$A = -s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 + s_\theta} \right)$
$s_e = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0{,}0$	$s_e = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0{,}0$

The plastic resistance is given by (P_n and p_n always positive outwards):

$$\frac{P_{nR}}{2\ell_m} + p_n = f_y \frac{t}{r}$$

B.3 Ring stiffened cylindrical shells

B.3.1 Ring stiffened Cylinder: Radial line load

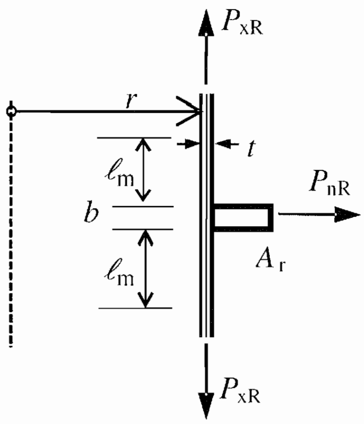


The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

$$\ell_m = \ell_o = 0{,}975\sqrt{rt}$$

B.3.2 Ring stiffened Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad \ell_o = 0{,}975\sqrt{rt}$$

Range of applicability:

$$-1 \leq s_x \leq +1$$

Dependent parameters:

If $P_n > 0$ then: $A = +s_x - 1{,}50$

If $P_n < 0$ then: $A = -s_x - 1{,}50$

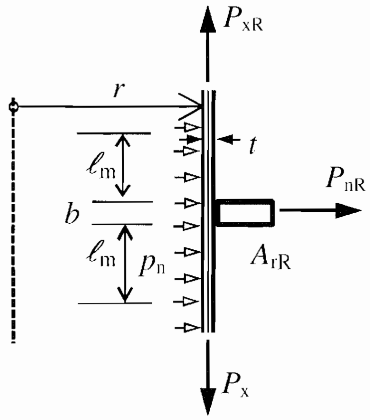
$$s_m = A + \sqrt{A^2 + 4(1 - s_x^2)}$$

If $s_x \neq 0$ then: $\ell_m = s_m \ell_o$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

B.3.3 Ring stiffened cylinder: Radial line load, constant internal pressure and axial load



Reference quantities:

$s_x = \frac{P_x}{f_y t}$ $s_x = \frac{P_x}{f_y t}$ $\qquad$ $s_\theta = \frac{p_n}{f_y} \cdot \frac{r}{t}$

$\ell_o = 0{,}975\sqrt{rt}$ $\qquad$ $s_e = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$

Range of applicability:

$-1 \le s_x \le +1$ $\qquad$ $-1 \le s_\theta \le +1$

Dependent parameters:

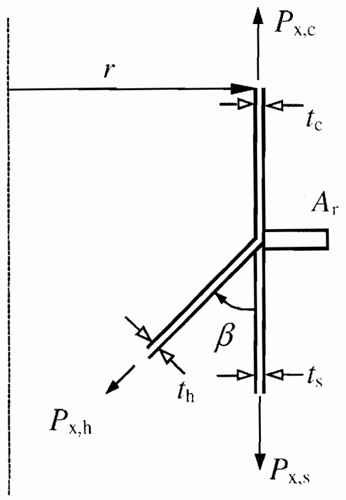
Outward directed ring load $P_n > 0$		Inward directed ring load $P_n < 0$	
Condition	Expressions	Condition	Expressions
$s_e < 1{,}00$ and $s_\theta \le 0{,}975$	$A = +s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 - s_\theta}\right)$	$s_e < 1{,}00$ and $s_\theta \ge -0{,}975$	$A = -s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 + s_\theta}\right)$
$s_e = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0{,}0$	$s_e = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0{,}0$

The plastic resistance is given by (P_n and p_n always positive outwards):

$$P_{nR} + p_n(b + 2\ell_m) = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r}\right)$$

B.4 Junctions between shells

B.4.1 Junction under meridional loading only (simplified)



Range of applicability:

$t_c^2 \leq t_s^2 + t_h^2$ $\qquad |P_{xs}| << t_s f_y$, $|P_{xh}| << t_h f_y$, and $|P_{xc}| << t_c f_y$

Dependent parameters:

$$\eta = \sqrt{\frac{t_c^2}{t_s^2 + t_h^2}} \qquad \psi_s = \psi_h = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$$

For the cylinder $\qquad \ell_{oc} = 0{,}975\sqrt{rt_c}$

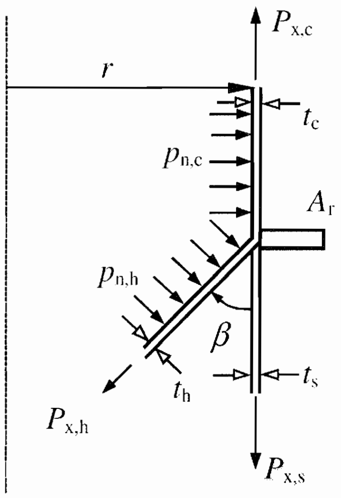
For the skirt $\qquad \ell_{os} = 0{,}975\ \psi_s\sqrt{rt_s}$

For the conical segment $\qquad \ell_{oh} = 0{,}975\ \psi_h\sqrt{\dfrac{rt_h}{\cos\beta}}$

The plastic resistance is given by:

$$P_{xhR}\ r\sin\beta = f_y\,(A_r + \ell_{oc}t_c + \ell_{os}t_s + \ell_{oh}t_h)$$

B.4.2 Junction under internal pressure and axial loading



Reference quantities:

$$s_{xc} = \frac{P_{xc}}{f_y t_c} \qquad s_{xs} = \frac{P_{xs}}{f_y t_s} \qquad s_{xh} = \frac{P_{xh}}{f_y t_h}$$

$$s_{\theta c} = \frac{p_{nc}}{f_y} \cdot \frac{r}{t_c} \qquad s_{\theta s} = 0 \qquad s_{\theta h} = \frac{p_{nh}}{f_y} \cdot \frac{r}{t_h \cdot \cos\beta}$$

for i = c, s, h in turn $\quad s_{ei} = \sqrt{s_{\theta i}^2 + s_{xi}^2 - s_{xi}s_{\theta i}}$

Range of applicability:

$$-1 \leq s_{xi} \leq +1 \qquad -1 \leq s_{\theta i} \leq +1$$

Equivalent thickness evaluation:

Lower plate group thicker $t_c^2 \leq t_s^2 + t_h^2$	Upper plate group thicker $t_c^2 > t_s^2 + t_h^2$
$\eta = \sqrt{\frac{t_c^2}{t_s^2 + t_h^2}}$	$\eta = \sqrt{\frac{t_s^2 + t_h^2}{t_c^2}}$
$\psi_c = 1{,}0$	$\psi_c = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$
$\psi_s = \psi_h = 0{,}7 + 0{,}6\eta^2 - 0{,}3\eta^3$	$\psi_s = \psi_h = 1{,}0$

Dependent parameters:

For the cylindrical segments $\quad \ell_{oi} = 0{,}975\ \psi_i \sqrt{r t_i}$

For the conical segment $\quad \ell_{oh} = 0{,}975\ \psi_h \sqrt{\frac{r t_i}{\cos\beta}}$

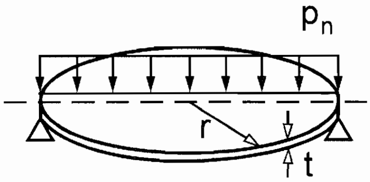
For each shell segment i separately	
Condition	Expressions
$s_{ei} < 1{,}00$ and $s_{\theta i} \geq -0{,}975$	$A_i = -s_{xi} + 2s_{\theta i} - 1{,}50$ $s_{mi} = A_i + \sqrt{A_i^2 + 4(1 - s_{ei}^2)}$ $\ell_{mi} = \ell_{oi}\ (\frac{s_{mi}}{1+s_{\theta i}})$
$s_{ei} = 1{,}00$	$\ell_{mi} = 0{,}0$
$s_{\theta i} < -0{,}975$	$\ell_{mi} = 0{,}0$

Plastic resistance is given by:

$$P_{xhR}\ r \sin\beta = f_y\,(A_r + \ell_{mc}t_c + \ell_{ms}t_s + \ell_{mh}t_h) + r\,(p_{nc}\ell_{mc} + p_{nh}\ell_{mh}\cos\beta)$$

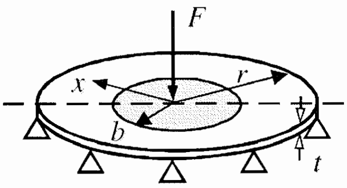
B.5 Circular plates with axisymmetric boundary conditions

B.5.1 Uniform load, simply supported boundary



$$p_{n,R} = 1{,}625\left(\frac{t}{r}\right)^2 f_y$$

B.5.2 Local distributed load, simply supported boundary



Uniform pressure p_n on circular patch of radius b

$F = p_n \pi b^2$

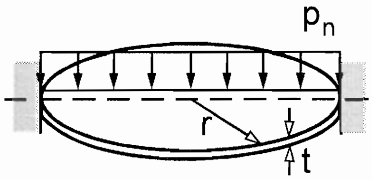
$$F_R = K\frac{\pi}{2}t^2 f_y$$

with

$$K = 1{,}0 + 1{,}10\frac{b}{r} + 1{,}15\left(\frac{b}{r}\right)^4 \quad \text{or} \quad K = \frac{1}{\sqrt{3}}\cdot\frac{b}{t}$$

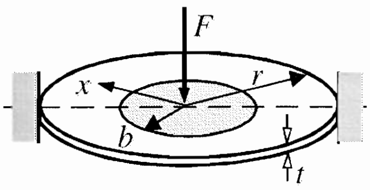
whichever is the lesser

B.5.3 Uniform load, clamped boundary



$$p_{n,R} = 3{,}125\left(\frac{t}{r}\right)^2 f_y$$

B.5.4 Local distributed load, clamped boundary



Uniform pressure p_n on circular patch of radius b

$F = p_n \pi b^2$

$$F_R = K\frac{\pi}{2}t^2 f_y$$

with $K = 1{,}40 + 2{,}85\frac{b}{r} + 2{,}0\left(\frac{b}{r}\right)^4$ or $K = \frac{1}{\sqrt{3}}\cdot\frac{b}{t}$

whichever is the lesser

ANNEX C (normative)
Expressions for linear elastic membrane and bending stresses

C.1 General

C.1.1 Action effects

The action effects calculated using the expressions in this annex may be assumed to provide characteristic values of the action effect when characteristic values of the actions, geometric parameters and material properties are adopted.

C.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman characters

b	radius at which local load on plate terminates
r	outside radius of circular plate
x	axial coordinate on cylinder or radial coordinate on circular plate

Greek symbols

$\sigma_{eq,m}$	von Mises equivalent stress associated with only membrane stress components
$\sigma_{eq,s}$	von Mises equivalent stress derived from surface stresses
σ_{MT}	reference stress derived from membrane theory
σ_{bx}	meridional bending stress
$\sigma_{b\theta}$	circumferential bending stress
σ_{sx}	meridional surface stress
$\sigma_{s\theta}$	circumferential surface stress
τ_{xn}	transverse shear stress associated with meridional bending

Subscripts

n	normal
r	relating to a ring
y	first yield value

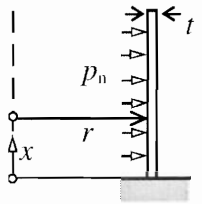
C.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 5.2.2.

(2) The term "clamped" should be taken to refer to BC1r and the term "pinned" to refer to BC2f.

C.2 Clamped base unstiffened cylindrical shells

C.2.1 Cylinder, clamped: uniform internal pressure

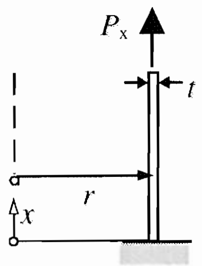


$$\sigma_{MT\theta} = p_n \frac{r}{t}$$

BC1r

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 1{,}816\ \sigma_{MT\theta}$	$+1{,}080\ \sigma_{MT\theta}$	$1{,}169\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}614\ \sigma_{MT\theta}$	$1{,}043\ \sigma_{MT\theta}$

C.2.2 Cylinder, clamped: axial loading

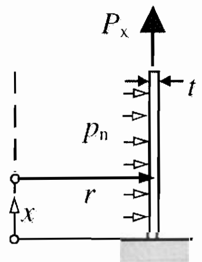


$$\sigma_{MTx} = \frac{P_x}{t}$$

BC1r

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$1{,}545\ \sigma_{MTx}$	$+0{,}455\ \sigma_{MTx}$	$0{,}351\sqrt{t/r}\ \sigma_{MTx}$	$1{,}373\ \sigma_{MTx}$	$1{,}000\ \sigma_{MTx}$

C.2.3 Cylinder, clamped: uniform internal pressure with axial loading



$$\sigma_{MT\theta} = p_n \frac{r}{t}$$

$$\sigma_{MTx} = \frac{P_x}{t}$$

BC1r

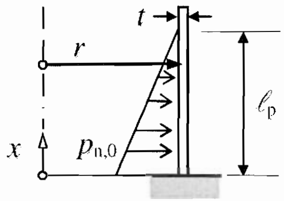
$$\text{Maximum } \sigma_{eq,m} = \sigma_{MT\theta} \sqrt{1 - \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right) + \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)^2}$$

$$\text{Maximum } \sigma_{eq,m} = k\ \sigma_{MT\theta}$$

$\left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$	−2,0	0	0	2,0
	Outer surface controls		Inner surface controls	
k	4,360	1,614	1,614	2,423

Linear interpolation may be used between values where the same surface controls

C.2.4 Cylinder, clamped: hydrostatic internal pressure



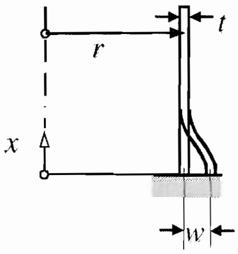
$$\sigma_{MT\theta} = p_{n0}\frac{r}{t}$$

BC1r

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau \sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

$\left(\frac{\sqrt{rt}}{\ell_p}\right)$	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
0	1,816	1,080	1,169	1,614	1,043
0,2	1,533	0,733	1,076	1,363	0,647

C.2.5 Cylinder, clamped: radial outward displacement

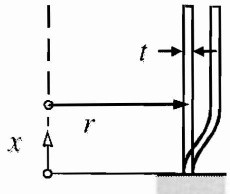


$$\sigma_{MT\theta} = \frac{wE}{r}$$

BC1r

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
1,816 $\sigma_{MT\theta}$	1,545 $\sigma_{MT\theta}$	1,169$\sqrt{t/r}\ \sigma_{MT\theta}$	2,081 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$

C.2.6 Cylinder, clamped: uniform temperature rise



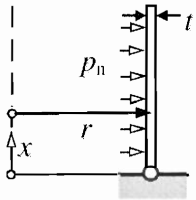
$$\sigma_{MT\theta} = \alpha E T$$

BC1r

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
1,816 $\sigma_{MT\theta}$	1,545 $\sigma_{MT\theta}$	1,169$\sqrt{t/r}\ \sigma_{MT\theta}$	2,081 $\sigma_{MT\theta}$	1,000 $\sigma_{MT\theta}$

C.3 Pinned base unstiffened cylindrical shells

C.3.1 Cylinder, pinned: uniform internal pressure

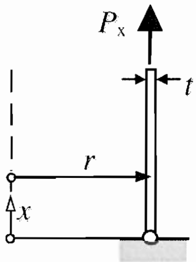


$$\sigma_{MT\theta} = p_n \frac{r}{t}$$

BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0{,}585\ \sigma_{MT\theta}$	$+1{,}125\ \sigma_{MT\theta}$	$0{,}583\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}126\ \sigma_{MT\theta}$	$1{,}067\ \sigma_{MT\theta}$

C.3.2 Cylinder, pinned: axial loading

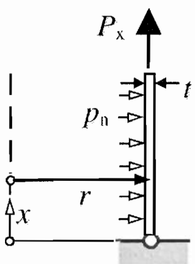


$$\sigma_{MTx} = \frac{P_x}{t}$$

BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$+1{,}176\ \sigma_{MTx}$	$+0{,}300\ \sigma_{MTx}$	$0{,}175\sqrt{t/r}\ \sigma_{MTx}$	$1{,}118\ \sigma_{MTx}$	$1{,}010\ \sigma_{MTx}$

C.3.3 Cylinder, pinned: uniform internal pressure with axial loading



$$\sigma_{MT\theta} = p_n \frac{r}{t}$$

$$\sigma_{MTx} = \frac{P_x}{t}$$

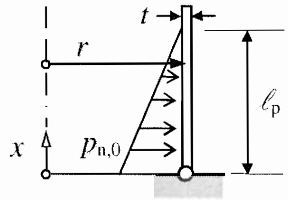
BC1f

Maximum $\sigma_{eq,m} = \sigma_{MTx}\sqrt{1 - \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right) + \left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)^2}$

Maximum $\sigma_{eq,s} = k\ \sigma_{MT\theta}$

$\left(\frac{\sigma_{MTx}}{\sigma_{MT\theta}}\right)$	−2,0	−1,0	−0,5	0,0	0,25	0,50	1,00	2,0
k	3,146	3,075	1,568	1,126	0,971	0,991	1,240	1,943

C.3.4 Cylinder, pinned: hydrostatic internal pressure



$$\sigma_{MT\theta} = p_{n0}\frac{r}{t}$$

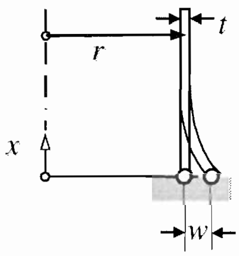
BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau \sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

$\left(\frac{\sqrt{rt}}{\ell_p}\right)$	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
0	0,585	1,125	0,583	1,126	1,067
0,2	0,585	0,873	0,583	0,919	0,759

Linear interpolation in $\left(\frac{\sqrt{rt}}{\ell_p}\right)$ may be used for different values of ℓ_p.

C.3.5 Cylinder, pinned: radial outward displacement

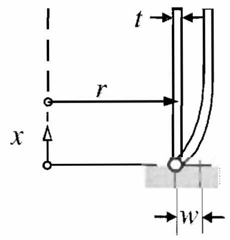


$$\sigma_{MT\theta} = \frac{wE}{r}$$

BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0{,}585\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$	$0{,}583\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$

C.3.6 Cylinder, pinned: uniform temperature rise

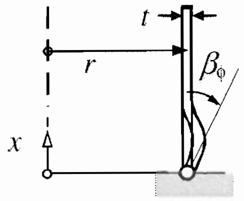


$$\sigma_{MT\theta} = \alpha E T$$
$$w = \alpha r T$$

BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0{,}585\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$	$0{,}583\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$	$1{,}000\ \sigma_{MT\theta}$

C.3.7 Cylinder, pinned: rotation of boundary



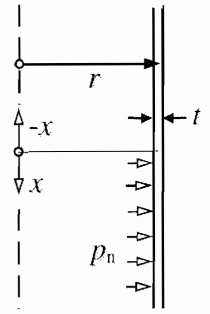
$$\sigma_{MT\theta} = E\sqrt{\frac{t}{r}} \cdot \beta_\Phi$$

BC1f

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 1{,}413\ \sigma_{MT\theta}$	$0{,}470\ \sigma_{MT\theta}$	$0{,}454\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}255\ \sigma_{MT\theta}$	$0{,}251\ \sigma_{MT\theta}$

C.4 Internal conditions in unstiffened cylindrical shells

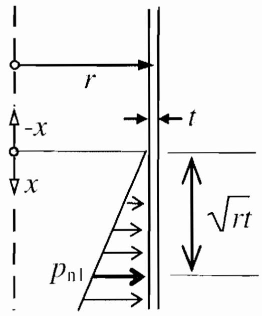
C.4.1 Cylinder: step change of internal pressure



$$\sigma_{MT\theta} = p_n \frac{r}{t}$$

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$\pm 0{,}293\ \sigma_{MT\theta}$	$1{,}062\ \sigma_{MT\theta}$	$0{,}467\sqrt{t/r}\ \sigma_{MT\theta}$	$1{,}056\ \sigma_{MT\theta}$	$1{,}033\ \sigma_{MT\theta}$

C.4.2 Cylinder: hydrostatic internal pressure termination



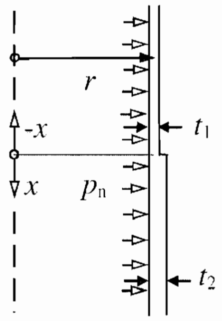
$$\sigma_{MT\theta} = p_{n1} \frac{r}{t}$$

p_{n1} is the pressure at a depth of $\sqrt{rt}$ below the surface

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau\sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
−1,060	0,510	0,160	1,005	0,275

C.4.3 Cylinder: step change of thickness



$$\sigma_{MT\theta} = p_n \frac{r}{t_1}$$

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau \sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

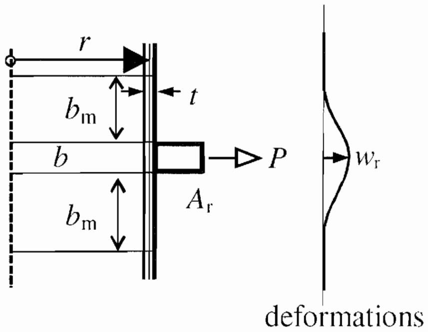
$\left(\frac{t_1}{t_2}\right)$	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
1,0	0,0	1,0	0,0	1,0	1,0
0,8	0,0256	1,010	0,179	1,009	0,895
0,667	0,0862	1,019	0,349	1,015	0,815
0,571	0,168	1,023	0,514	1,019	0,750
0,5	0,260	1,027	0,673	1,023	0,694

C.5 Ring stiffener on cylindrical shell

C.5.1 Ring stiffened cylinder: radial force on ring

The stresses in the shell should be determined using the calculated value of w from this clause introduced into the expressions given in C.2.5.

Where there is a change in the shell thickness at the ring, the method set out in 8.2.2 of EN 1993-4-1 should be used.



$$w = w_r$$

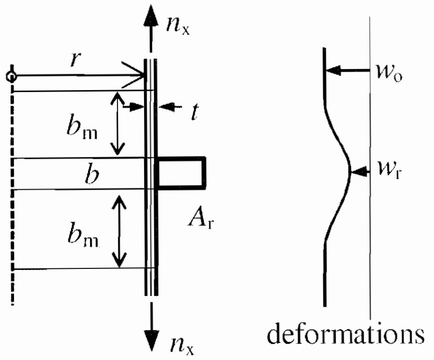
$$b_m = 0{,}778\sqrt{rt}$$

$$\left(\frac{wE}{r}\right) = \frac{P \cdot r}{A_r + (b + 2b_m)t}$$

$$\sigma_{\theta r} = \frac{P \cdot r}{A_r + (b + 2b_m)t}$$

C.5.2 Ring stiffened cylinder: axial loading

The stresses in the shell should be determined using the calculated value of w from this clause introduced into the expressions given in C.2.5 and C.2.2.



$$\sigma_{MTx} = \frac{n_x}{t}$$

$$w = w_r - w_o$$

$$w_0 = -\nu\sigma_{MTx}\frac{r}{E}$$

$$b_m = 0{,}778\sqrt{rt}$$

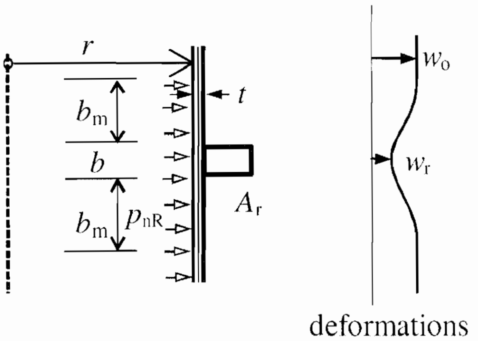
$$w_r = w_0\frac{(b+2b_m)t}{A_r+(b+2b_m)t}$$

$$w = -w_0\frac{A_r}{A_r+(b+2b_m)t}$$

$$\sigma_{\theta r} = E\frac{w_r}{r}$$

C.5.3 Ring stiffened cylinder: uniform internal pressure

The stresses in the shell should be determined using the calculated value of w from this clause introduced into the expressions given in C.2.5 and C.2.1.



$$\sigma_{MT\theta} = \frac{p_n r}{t}$$

$$w = w_r - w_o$$

$$w_0 = \sigma_{MT\theta}\frac{r}{E}$$

$$b_m = 0{,}778\sqrt{rt}$$

$$w_r = w_o(1-\kappa)$$

$$w = -w_o\kappa$$

$$\kappa = \frac{A_r}{A_r+(b+2b_m)t}$$

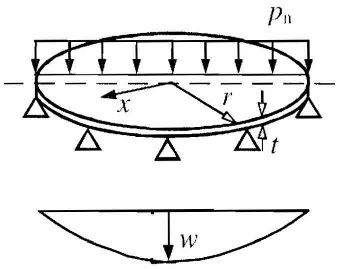
$$\sigma_{\theta r} = E\frac{w_r}{r}$$

Maximum σ_{sx}	Maximum $\sigma_{s\theta}$	Maximum τ_{xn}	Maximum $\sigma_{eq,s}$	Maximum $\sigma_{eq,m}$
$k_x\ \sigma_{MT\theta}$	$k_\theta\ \sigma_{MT\theta}$	$k_\tau\sqrt{t/r}\ \sigma_{MT\theta}$	$k_{eq,s}\ \sigma_{MT\theta}$	$k_{eq,m}\ \sigma_{MT\theta}$

κ	k_x	k_θ	k_τ	$k_{eq,s}$	$k_{eq,m}$
1,0	1,816	1,080	1,169	1,614	1,043
0,75	1,312	1,060	0,877	1,290	1,032
0,50	0,908	1,040	0,585	1,014	1,021
0,0	0,0	1,000	0,0	1,000	1,000

C.6 Circular plates with axisymmetric boundary conditions

C.6.1 Plate with simply supported boundary: uniform load



deflected shape

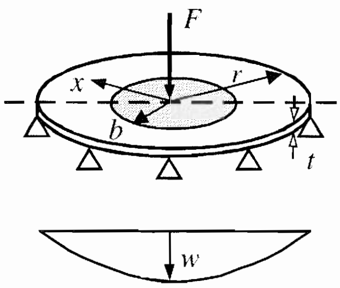
$$w = 0{,}696 \frac{p_n r^4}{E t^3}$$

$$\max.\ \sigma_{xb} = 1{,}238\, p_n \left(\frac{r}{t}\right)^2$$

$$\max.\ \sigma_{\theta b} = 1{,}238\, p_n \left(\frac{r}{t}\right)^2$$

$$p_{n,y} = 0{,}808 \left(\frac{t}{r}\right)^2 f_y$$

C.6.2 Plate with local distributed load: simply supported boundary



deflected shape

Uniform pressure p_n on circular patch of radius b

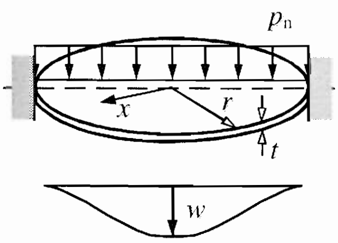
$F = p_n \pi b^2$ $\qquad b < 0{,}2\, r$

$$w = 0{,}606 \frac{F r^2}{E t^3}$$

$$\max.\ \sigma_{xb} = \max.\ \sigma_{\theta b} = 0{,}621 \frac{F}{t^2} \left(\ln \frac{b}{r} + 0{,}769\right)$$

$$F_y = 1{,}611 \frac{t^2}{\left(\ln \frac{b}{r} + 0{,}769\right)} f_y$$

C.6.3 Plate with fixed boundary: uniform load



deflected shape

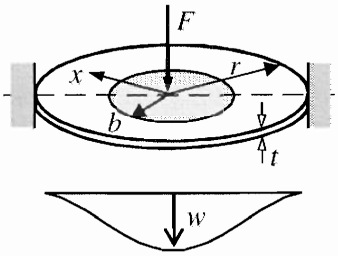
$$w = 0{,}171 \frac{p_n r^4}{E t^3}$$

$$\sigma_0 = p_n \left(\frac{r}{t}\right)^2$$

$$p_{n,y} = 1{,}50 \left(\frac{t}{r}\right)^2 f_y \qquad \text{(at edge)}$$

Maximum σ_{bx} at centre	Maximum $\sigma_{b\theta}$ at centre	Maximum σ_{eq} at centre	Maximum σ_{bx} at edge	Maximum $\sigma_{b\theta}$ at edge	Maximum σ_{eq} at edge
0,488 σ_0	0,488 σ_0	0,488 σ_0	0,75 σ_0	0,225 σ_0	0,667 σ_0

C.6.4 Plate with fixed boundary: local distributed load



deflected shape

Uniform pressure p_n on circular patch of radius b

$F = p_n \pi b^2$ $\qquad b < 0{,}2\, r$

$$w = 0{,}217 \frac{Fr^2}{Et^3}$$

$$\sigma_0 = \frac{F}{t^2} \qquad F_y = 1{,}611 \frac{t^2}{(\ln \frac{b}{r})} f_y \quad \text{at centre}$$

Maximum σ_{bx} at centre	Maximum $\sigma_{b\theta}$ at centre	Maximum σ_{eq} at centre	Maximum σ_{bx} at edge	Maximum $\sigma_{b\theta}$ at edge	Maximum σ_{eq} at edge
$0{,}621(\ln \frac{b}{r})\sigma_0$	$0{,}621(\ln \frac{b}{r})\sigma_0$	$0{,}621(\ln \frac{b}{r})\sigma_0$	0,477 σ_0	0,143 σ_0	0,424 σ_0

ANNEX D (normative)
Expressions for buckling stress ⒜ *text deleted* ⒜

D.1 Unstiffened cylindrical shells of constant wall thickness

D.1.1 Notation and boundary conditions

(1) Geometrical quantities

ℓ	cylinder length between defined boundaries
r	radius of cylinder middle surface
t	thickness of shell
Δw_k	characteristic imperfection amplitude

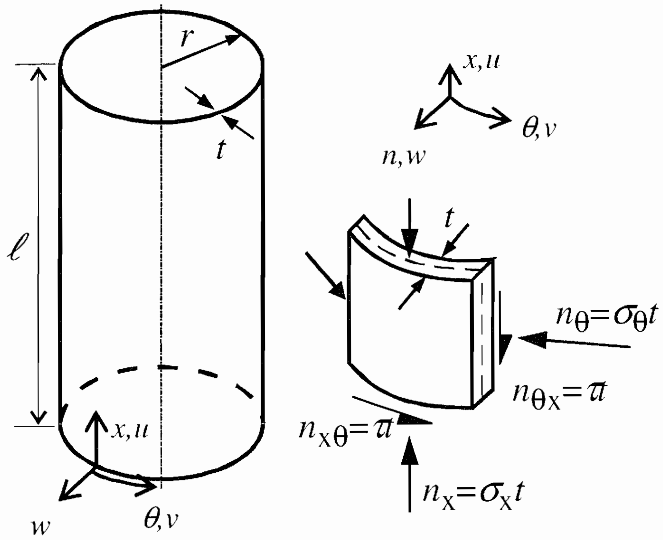


Figure D.1: Cylinder geometry, membrane stresses and stress resultants

(2) The relevant boundary conditions are set out in 2.3, 5.2.2 and 8.3.

D.1.2 Meridional (axial) compression

D.1.2.1 Critical meridional buckling stresses

(1) The following expressions may only be used for shells with boundary conditions BC 1 or BC 2 at both edges.

(2) The length of the shell segment is characterised in terms of the dimensionless length parameter ω:

$$\omega = \frac{\ell}{r}\sqrt{\frac{r}{t}} = \frac{\ell}{\sqrt{rt}} \qquad \text{... (D.1)}$$

(3) The elastic critical meridional buckling stress, using a value of C_x from (4), (5) or (6), should be obtained from:

$$\sigma_{x,Rcr} = 0,605 E C_x \frac{t}{r} \qquad \text{... (D.2)}$$

(4) For medium-length cylinders, which are defined by:

$$1,7 \leq \omega \leq 0,5 \frac{r}{t} \qquad \text{... (D.3)}$$

the factor C_x should be taken as:

$$C_x = 1,0 \qquad \text{... (D.4)}$$

(5) For short cylinders, which are defined by:

$$\omega \leq 1,7 \qquad \text{... (D.5)}$$

the factor C_x may be taken as:

$$C_x = 1,36 - \frac{1,83}{\omega} + \frac{2,07}{\omega^2} \qquad \text{... (D.6)}$$

(6) For long cylinders, which are defined by:

$$\omega > 0,5 \frac{r}{t} \qquad \text{... (D.7)}$$

the factor C_x should be found as:

$$C_x = C_{x,N} \qquad \text{... (D.8)}$$

in which $C_{x,N}$ is the greater of:

$$C_{x,N} = 1 + \frac{0,2}{C_{xb}} \left[1 - 2\omega \frac{t}{r} \right] \qquad \text{... (D.9)}$$

and

$$C_{x,N} = 0,60 \qquad \text{... (D.10)}$$

where C_{xb} is a parameter depending on the boundary conditions and being taken from table D.1.

Table D.1: Parameter C_{xb} for the effect of boundary conditions on the elastic critical meridional buckling stress in long cylinders

Case	Cylinder end	Boundary condition	C_{xb}
1	end 1 end 2	BC 1 BC 1	6
2	end 1 end 2	BC 1 BC 2	3
3	end 1 end 2	BC 2 BC 2	1

(7) For long cylinders as defined in (6) that satisfy the additional conditions:

$$\frac{r}{t} \leq 150 \text{ and } \omega \leq 6\left(\frac{r}{t}\right) \text{ and } 500 \leq \frac{E}{f_{y,k}} \leq 1000 \qquad \text{... (D.11)}$$

the factor C_x may alternatively be obtained from:

$$C_x = C_{x,N}\left(\frac{\sigma_{xE,N}}{\sigma_{xE}}\right)+\left(\frac{\sigma_{xE,M}}{\sigma_{xE}}\right) \qquad \text{... (D.12)}$$

where:

σ_{xE} is the design value of the meridional stress $\sigma_{x,Ed}$

$\sigma_{xE,N}$ is the component of $\sigma_{x,Ed}$ that derives from axial compression (circumferentially uniform component)

$\sigma_{xE,M}$ is the component of $\sigma_{x,Ed}$ that derives from tubular global bending (peak value of the circumferentially varying component)

The following simpler expression may also be used in place of expression (D.12):

$$C_x = 0{,}60 + 0{,}40\left(\frac{\sigma_{xE,M}}{\sigma_{xE}}\right) \qquad \text{... (D.13)}$$

D.1.2.2 Meridional buckling parameters

(1) The meridional elastic imperfection reduction factor α_x should be obtained from:

$$\alpha_x = \frac{0{,}62}{1+1{,}91(\Delta w_k / t)^{1{,}44}} \qquad \text{... (D.14)}$$

where Δw_k is the characteristic imperfection amplitude:

$$\Delta w_k = \frac{1}{Q}\sqrt{\frac{r}{t}} \cdot t \qquad \text{... (D.15)}$$

where Q is the meridional compression fabrication quality parameter.

(2) The fabrication quality parameter Q should be taken from table D.2 for the specified fabrication tolerance quality class.

Table D.2: Values of fabrication quality parameter *Q*

Fabrication tolerance quality class	Description	Q
Class A	Excellent	40
Class B	High	25
Class C	Normal	16

(3) The meridional squash limit slenderness $\overline{\lambda}_{x0}$, the plastic range factor β, and the interaction exponent η should be taken as:

$$\overline{\lambda}_{x0} = 0{,}20 \qquad \beta = 0{,}60 \qquad \eta = 1{,}0 \qquad \text{... (D.16)}$$

(4) For long cylinders that satisfy the special conditions of D.1.2.1 (7), the meridional squash limit slenderness $\overline{\lambda}_{x0}$ may be obtained from:

$$\overline{\lambda}_{x0} = 0{,}20 + 0{,}10\left(\frac{\sigma_{xE,M}}{\sigma_{xE}}\right) \qquad \text{... (D.17)}$$

where:

σ_{xE} is the design value of the meridional stress $\sigma_{x,Ed}$

$\sigma_{xE.M}$ is the component of $\sigma_{x,Ed}$ that derives from tubular global bending (peak value of the circumferentially varying component)

(5) Cylinders need not be checked against meridional shell buckling if they satisfy:

$$\frac{r}{t} \le 0{,}03\frac{E}{f_{yk}} \qquad \text{... (D.18)}$$

D.1.3 Circumferential (hoop) compression

D.1.3.1 Critical circumferential buckling stresses

(1) The following expressions may be applied to shells with all boundary conditions.

(2) The length of the shell segment should be characterised in terms of the dimensionless length parameter ω:

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} \qquad \text{... (D.19)}$$

(3) For medium-length cylinders, which are defined by:

$$20 \le \frac{\omega}{C_\theta} \le 1{,}63\frac{r}{t} \qquad \text{... (D.20)}$$

the elastic critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,Rcr} = 0{,}92E\left(\frac{C_\theta}{\omega}\right)\left(\frac{t}{r}\right) \qquad \text{... (D.21)}$$

(4) The factor C_θ should be taken from table D.3, with a value that depends on the boundary conditions, see 5.2.2 and 8.3.

(5) For short cylinders, which are defined by:

$$\frac{\omega}{C_\theta} < 20 \qquad \text{... (D.22)}$$

the elastic critical circumferential buckling stress should be obtained instead from:

$$\sigma_{\theta,Rcr} = 0{,}92E\left(\frac{C_{\theta s}}{\omega}\right)\left(\frac{t}{r}\right) \qquad \text{... (D.23)}$$

(6) The factor $C_{\theta s}$ should be taken from table D.4, with a value that depends on the boundary conditions, see 5.2.2 and 8.3:

Table D.3: External pressure buckling factors for medium-length cylinders C_θ

Case	Cylinder end	Boundary condition	Value of C_θ
1	end 1 end 2	BC 1 BC 1	1,5
2	end 1 end 2	BC 1 BC 2	1,25
3	end 1 end 2	BC 2 BC 2	1,0
4	end 1 end 2	BC 1 BC 3	0,6
5	end 1 end 2	BC2 BC3	0
6	end 1 end 2	BC 3 BC 3	0

Table D.4: External pressure buckling factors for short cylinders $C_{\theta s}$

Case	Cylinder end	Boundary condition	$C_{\theta s}$
1	end 1 end 2	BC 1 BC 1	$1{,}5+\frac{10}{\omega^2}-\frac{5}{\omega^3}$
2	end 1 end 2	BC 1 BC 2	$1{,}25+\frac{8}{\omega^2}-\frac{4}{\omega^3}$
3	end 1 end 2	BC 2 BC 2	$1{,}0+\frac{3}{\omega^{1{,}35}}$
4	end 1 end 2	BC 1 BC 3	$0{,}6+\frac{1}{\omega^2}-\frac{0{,}3}{\omega^3}$
where $\omega=\frac{l}{\sqrt{rt}}$			

(7) For long cylinders, which are defined by:

$$\frac{\omega}{C_\theta} > 1{,}63\frac{r}{t} \qquad \text{... (D.24)}$$

the elastic critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,Rcr} = E\left(\frac{t}{r}\right)^2\left[0,275 + 2,03\left(\frac{C_\theta}{\omega}\cdot\frac{r}{t}\right)^4\right] \quad \text{... (D.25)}$$

D.1.3.2 Circumferential buckling parameters

(1) The circumferential elastic imperfection reduction factor should be taken from table D.5 for the specified fabrication tolerance quality class.

Table D.5 : Values of α_θ based on fabrication quality

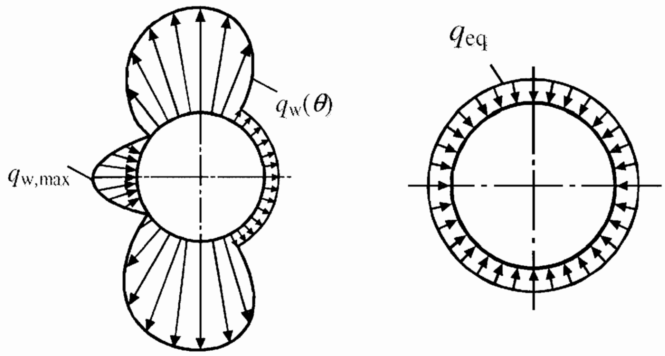
Fabrication tolerance quality class	Description	α_θ
Class A	Excellent	0,75
Class B	High	0,65
Class C	Normal	0,50

(2) The circumferential squash limit slenderness $\bar{\lambda}_{\theta 0}$, the plastic range factor β, and the interaction exponent η should be taken as:

$$\bar{\lambda}_{\theta 0} = 0,40 \qquad \beta = 0,60 \qquad \eta = 1,0 \quad \text{... (D.26)}$$

(3) Cylinders need not be checked against circumferential shell buckling if they satisfy:

$$\frac{r}{t} \le 0,21\sqrt{\frac{E}{f_{yk}}} \quad \text{... (D.27)}$$



a) wind pressure distribution around shell circumference

b) equivalent axisymmetric pressure distribution

Figure D.2: Transformation of typical wind external pressure load distribution

(4) The non-uniform distribution of pressure q_w resulting from external wind loading on cylinders (see figure D.2) may, for the purpose of shell buckling design, be substituted by an equivalent uniform external pressure:

$$q_{eq} = k_w \; q_{w,max} \qquad \text{... (D.28)}$$

where $q_{w,max}$ is the maximum wind pressure, and k_w should be found as follows:

$$k_w = 0{,}46\left(1 + 0{,}1\sqrt{\frac{C_\theta}{\omega} \cdot \frac{r}{t}}\right) \qquad \text{... (D.29)}$$

with the value of k_w not outside the range $0{,}65 \leq k_w \leq 1$, and with C_θ taken from table D.3 according to the boundary conditions.

(5) The circumferential design stress to be introduced into 8.5 follows from:

$$\sigma_{\theta,Ed} = (q_{eq} + q_s)\left(\frac{r}{t}\right) \qquad \text{... (D.30)}$$

where q_s is the internal suction caused by venting, internal partial vacuum or other phenomena.

D.1.4 Shear

D.1.4.1 Critical shear buckling stresses

(1) The following expressions should be applied only to shells with boundary conditions BC1 or BC2 at both edges.

(2) The length of the shell segment should be characterised in terms of the dimensionless length parameter ω:

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} \qquad \text{... (D.31)}$$

(3) The elastic critical shear buckling stress should be obtained from:

$$\tau_{x\theta,Rcr} = 0{,}75 E C_\tau \sqrt{\frac{1}{\omega}}\left(\frac{t}{r}\right) \qquad \text{... (D.32)}$$

(4) For medium-length cylinders, which are defined by:

$$10 \leq \omega \leq 8{,}7\frac{r}{t} \qquad \text{... (D.33)}$$

the factor C_τ may be found as:

$$C_\tau = 1{,}0 \qquad \text{... (D.34)}$$

(5) For short cylinders, which are defined by:

$$\omega < 10 \qquad \text{... (D.35)}$$

the factor C_τ may be obtained from:

$$C_\tau = \sqrt{1 + \frac{42}{\omega^3}} \qquad \text{... (D.36)}$$

(6) For long cylinders, which are defined by:

$$\omega > 8{,}7\frac{r}{t} \qquad \text{... (D.37)}$$

the factor C_τ may be obtained from:

$$C_\tau = \frac{1}{3}\sqrt{\omega\frac{t}{r}} \qquad \text{... (D.38)}$$

D.1.4.2 Shear buckling parameters

(1) The shear elastic imperfection reduction factor should be taken from table D.6 for the specified fabrication tolerance quality class.

Table D.6: Values of α_τ based on fabrication quality

Fabrication tolerance quality class	Description	α_τ
Class A	Excellent	0,75
Class B	High	0,65
Class C	Normal	0,50

(2) The shear squash limit slenderness $\bar{\lambda}_{\tau 0}$, the plastic range factor β, and the interaction exponent η should be taken as:

$$\bar{\lambda}_{\tau 0} = 0{,}40 \qquad \beta = 0{,}60 \qquad \eta = 1{,}0 \qquad \text{... (D.39)}$$

(3) Cylinders need not be checked against shear shell buckling if they satisfy:

$$\frac{r}{t} \leq 0{,}16\left[\frac{E}{f_{yk}}\right]^{0.67} \qquad \text{... (D.40)}$$

D.1.5 Meridional (axial) compression with coexistent internal pressure

D.1.5.1 Pressurised critical meridional buckling stress

(1) The elastic critical meridional buckling stress $\sigma_{x,\mathrm{Rcr}}$ may be assumed to be unaffected by the presence of internal pressure and may be obtained as specified in D.1.2.1.

D.1.5.2 Pressurised meridional buckling parameters

(1) The pressurised meridional buckling stress should be verified analogously to the unpressurised meridional buckling stress as specified in 8.5 and D.1.2.2. However, the unpressurised elastic

imperfection reduction factor α_x should be replaced by the pressurised elastic imperfection reduction factor α_{xp}.

(2) The pressurised elastic imperfection reduction factor α_{xp} should be taken as the smaller of the two following values:

α_{xpe} is a factor covering pressure-induced elastic stabilisation;

α_{xpp} is a factor covering pressure-induced plastic destabilisation

(3) The factor α_{xpe} should be obtained from:

$$\alpha_{xpe} = \alpha_x + (1-\alpha_x)\left[\frac{\overline{p}_s}{\overline{p}_s + 0{,}3/\alpha_x^{0,5}}\right] \qquad \text{... (D.41)}$$

$$\overline{p}_s = \left(\frac{p_s}{\sigma_{x,Rcr}}\right)\left(\frac{r}{t}\right) \qquad \text{... (D.42)}$$

where:

p_s is the smallest design value of local internal pressure at the location of the point being assessed, guaranteed to coexist with the meridional compression,

α_x is the unpressurised meridional elastic imperfection reduction factor according to D.1.2.2, and

$\sigma_{x,Rcr}$ is the elastic critical meridional buckling stress according to D.1.2.1 (3).

(4) The factor α_{xpe} should not be applied to cylinders that are long according to D.1.2.1 (6). In addition, it should not be applied unless one of the following two conditions are met:

the cylinder is medium-length according to D.1.2.1 (4);
the cylinder is short according to D.1.2.1 (5) and $C_x = 1$ has been adopted in D.1.2.1 (3).

(5) The factor α_{xpp} should be obtained from:

$$\alpha_{xpp} = \left\{1 - \left(\frac{\overline{p}_g}{\overline{\lambda}_x^2}\right)^2\right\}\left[1 - \frac{1}{1{,}12 + s^{3/2}}\right]\left[\frac{s^2 + 1{,}21\overline{\lambda}_x^2}{s(s+1)}\right] \qquad \text{... (D.43)}$$

$$\overline{p}_g = \left(\frac{p_g}{\sigma_{x,Rcr}}\right)\left(\frac{r}{t}\right) \qquad \text{... (D.44)}$$

$$s = \frac{1}{400} \cdot \frac{r}{t} \qquad \text{... (D.45)}$$

where:

p_g is the largest design value of local internal pressure at the location of the point being assessed that can coexist with the meridional compression;

$\overline{\lambda}_x$ is the dimensionless shell slenderness parameter according to 8.5.2 (6);

$\sigma_{x,Rcr}$ is the elastic critical meridional buckling stress according to D.1.2.1 (3).

D.1.6 Combinations of meridional (axial) compression, circumferential (hoop) compression and shear

(1) The buckling interaction parameters to be used in 8.5.3 (3) may be obtained from:

$$k_x = 1{,}25 + 0{,}75\,\chi_x \qquad \text{... (D.46)}$$

$$k_\theta = 1{,}25 + 0{,}75\,\chi_\theta \qquad \text{... (D.47)}$$

$$k_\tau = 1{,}75 + 0{,}25\,\chi_\tau \qquad \text{... (D.48)}$$

$$k_i = (\chi_x\,\chi_\theta)^2 \qquad \text{... (D.49)}$$

where:

χ_x, χ_θ, χ_τ are the buckling reduction factors defined in 8.5.2, using the buckling parameters given in D.1.2 to D.1.4.

(2) The three membrane stress components should be deemed to interact in combination at any point in the shell, except those adjacent to the boundaries. The buckling interaction check may be omitted for all points that lie within the boundary zone length ℓ_R adjacent to either end of the cylindrical segment. The value of ℓ_R is the smaller of:

$$\ell_R = 0{,}1L \qquad \text{... (D.50)}$$

and

$$\ell_R \leq 0{,}16\,r\sqrt{r/t} \qquad \text{... (D.51)}$$

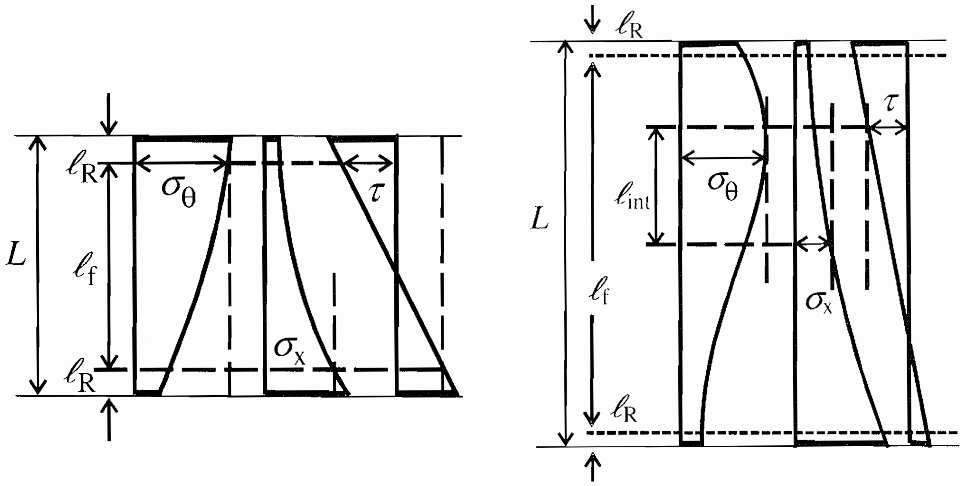
(3) Where checks of the buckling interaction at all points is found to be onerous, the following provisions of (4) and (5) permit a simpler conservative assessment. If the maximum value of any of the buckling-relevant membrane stresses in a cylindrical shell occurs in a boundary zone of length ℓ_R adjacent to either end of the cylinder, the interaction check of 8.5.3 (3) may be undertaken using the values defined in (4).

(4) Where the conditions of (3) are met, the maximum value of each of the buckling-relevant membrane stresses occurring within the free length ℓ_f (that is, outside the boundary zones, see figure D.3a) may be used in the interaction check of 8.5.3 (3), where:

$$\ell_f = L - 2\ell_R \qquad \text{... (D.52)}$$

(5) For long cylinders as defined in D.1.2.1 (6), the interaction-relevant groups introduced into the interaction check may be restricted further than the provisions of paragraphs (3) and (4). The stresses deemed to be in interaction-relevant groups may then be restricted to any section of length ℓ_{int} falling within the free remaining length ℓ_f for the interaction check (see figure D.3b), where:

$$\ell_{int} = 1{,}3\, r\sqrt{r/t} \qquad \text{... (D.53)}$$



a) in a short cylinder b) in a long cylinder

Figure D.3: Examples of interaction-relevant groups of membrane stress components

(6) If (3)-(5) above do not provide specific provisions for defining the relative locations or separations of interaction-relevant groups of membrane stress components, and a simple conservative treatment is still required, the maximum value of each membrane stress, irrespective of location in the shell, may be adopted into expression (8.19).

D.2 Unstiffened cylindrical shells of stepwise variable wall thickness

D.2.1 General

D.2.1.1 Notation and boundary conditions

(1) In this clause the following notation is used:

- L overall cylinder length
- r radius of cylinder middle surface
- j an integer index denoting the individual cylinder sections with constant wall thickness (from $j = 1$ to $j = n$)
- t_j the constant wall thickness of section j of the cylinder
- ℓ_j the length of section j of the cylinder

(2) The following expressions may only be used for shells with boundary conditions BC 1 or BC 2 at both edges (see 5.2.2 and 8.3), with no distinction made between them.

D.2.1.2 Geometry and joint offsets

(1) Provided that the wall thickness of the cylinder increases progressively stepwise from top to bottom (see figure D.5a), the procedures given in this clause D.2 may be used.

(2) Intended offsets e_0 between plates of adjacent sections (see figure D.4) may be treated as covered by the following expressions provided that the intended value e_0 is less than the permissible value $e_{0,p}$ which should be taken as the smaller of:

$$e_{0,p} = 0{,}5\,(t_{max} - t_{min}) \qquad \text{... (D.54)}$$

and

$$e_{0,p} = 0{,}5\,t_{min} \qquad \text{... (D.55)}$$

where:

t_{max} is the thickness of the thicker plate at the joint;
t_{min} is the thickness of the thinner plate at the joint.

(3) For cylinders with permissible intended offsets between plates of adjacent sections according to (2), the radius r may be taken as the mean value of all sections.

(4) For cylinders with overlapping joints (lap joints), the provisions for lap-jointed construction given in D.3 below should be used.

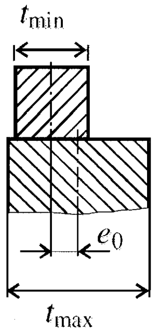


Figure D.4: Intended offset e_0 in a butt-jointed shell

D.2.2 Meridional (axial) compression

(1) Each cylinder section j of length ℓ_j should be treated as an equivalent cylinder of overall length $\ell = L$ and of uniform wall thickness $t = t_j$ according to D.1.2.

(2) For long equivalent cylinders, as governed by D.1.2.1 (6), the parameter C_{xb} should be conservatively taken as $C_{xb} = 1$, unless a better value is justified by more rigorous analysis.

D.2.3 Circumferential (hoop) compression

D.2.3.1 Critical circumferential buckling stresses

(1) If the cylinder consists of three sections with different wall thickness, the procedure of (4) to (7) should be applied to the real sections a, b and c, see figure D.5b.

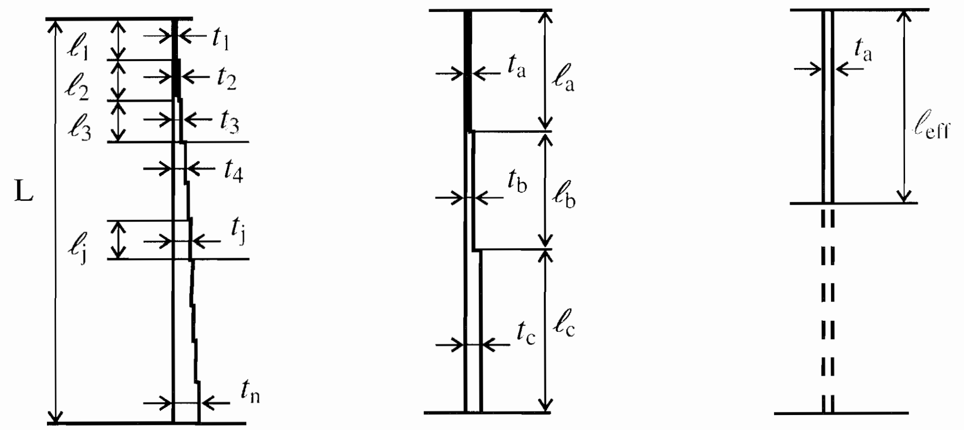
(2) If the cylinder consists of only one section (i.e. constant wall thickness), D.1 should be applied.

(3) If the cylinder consists of two sections of different wall thickness, the procedure of (4) to (7) should be applied, treating two of the three fictitious sections, a and b, as being of the same thickness.

(4) If the cylinder consists of more than three sections with different wall thicknesses (see figure D.5a), it should first be replaced by an equivalent cylinder comprising three sections a, b and c (see figure D.5b). The length of its upper section, ℓ_a, should extend to the upper edge of the first section that has a wall thickness greater than 1,5 times the smallest wall thickness t_1, but should not comprise more than half the total length L of the cylinder. The length of the two other sections ℓ_b and ℓ_c should be obtained as follows:

$$\ell_b = \ell_a \text{ and } \ell_c = L - 2\ell_a, \quad \text{if } \ell_a \leq L/3 \qquad \text{... (D.56)}$$

$$\ell_b = \ell_c = 0{,}5\,(L - \ell_a), \quad \text{if } L/3 < \ell_a \leq L/2 \qquad \text{... (D.57)}$$



(a) Cylinder of stepwise variable wall thickness

(b) Equivalent cylinder comprising three sections

(c) Equivalent single cylinder with uniform wall thickness

Figure D.5: Transformation of stepped cylinder into equivalent cylinder

(5) The fictitious wall thicknesses t_a, t_b and t_c of the three sections should be determined as the weighted average of the wall thickness over each of the three fictitious sections:

$$t_a = \frac{1}{\ell_a} \sum_a \ell_j t_j \qquad \text{... (D.58)}$$

$$t_b = \frac{1}{\ell_b} \sum_b \ell_j t_j \qquad \text{... (D.59)}$$

$$t_c = \frac{1}{\ell_c} \sum_c \ell_j t_j \qquad \text{... (D.60)}$$

(6) The three-section-cylinder (i.e. the equivalent one or the real one respectively) should be replaced by an equivalent single cylinder of effective length ℓ_{eff} and of uniform wall thickness $t = t_a$, see figure D.5c.

The effective length should be determined from:

$$\ell_{eff} = \ell_a / \kappa \qquad \text{... (D.61)}$$

in which κ is a dimensionless factor obtained from figure D.6.

(7) For cylinder sections of moderate or short length, the elastic critical circumferential buckling stress of each cylinder section j of the original cylinder of stepwise variable wall thickness should be determined from:

$$\sigma_{\theta,Rcr,j} = \left(\frac{t_a}{t_j}\right)\sigma_{\theta,Rcr,eff} \qquad \text{... (D.62)}$$

where $\sigma_{\theta,\mathrm{Rcr,eff}}$ is the elastic critical circumferential buckling stress derived from D.1.3.1 (3), D.1.3.1 (5) or D.1.3.1 (7), as appropriate, of the equivalent single cylinder of length ℓ_{eff} according to paragraph (6). The factor C_θ in these expressions should be given the value $C_\theta = 1{,}0$.

NOTE: Expression D.62 may seem strange in that the resistance appears to be higher in thinner plates. The reason is that the whole cylinder bifurcates at a single critical external pressure, and expression D.62 gives the membrane stress in each course at that instant. Since the external pressure is axially uniform, these stress values are smaller in the thicker courses. It should be noted that the design membrane circumferential stress, with which the resistance stresses will be compared in a design check, is also smaller in the thicker courses (see figure D.7). If a stepped cylinder is elastic and under uniform external pressure, the ratio of the design membrane circumferential stress to the design resistance stress is constant throughout all courses.

(8) The length of the shell segment is characterised in terms of the dimensionless length parameter ω_j:

$$\omega_j = \frac{\ell_j}{r}\sqrt{\frac{r}{t_j}} = \frac{\ell_j}{\sqrt{r t_j}} \qquad \text{... (D.63)}$$

(9) Where the cylinder section j is long, a second additional assessment of the buckling stress should be made. The smaller of the two values derived from (7) and (10) should be used for the buckling design check of the cylinder section j.

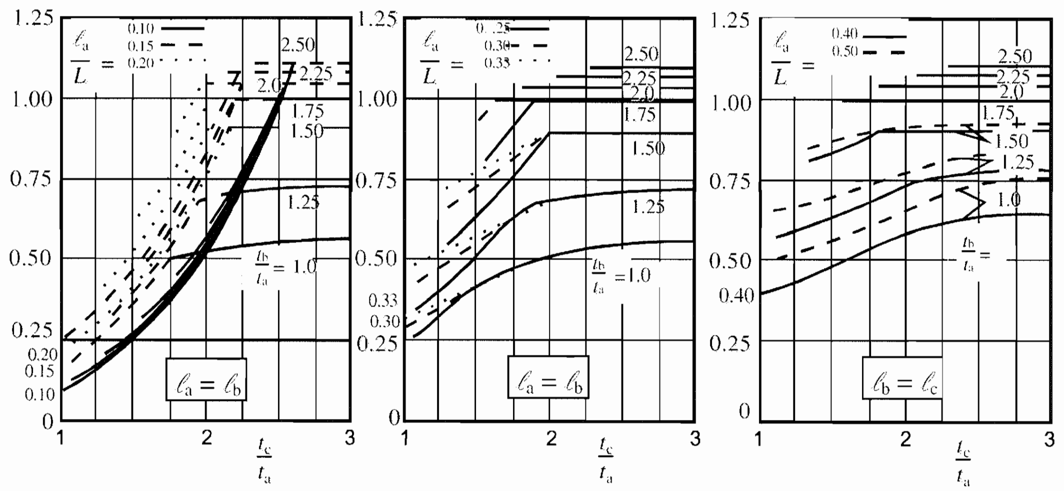


Figure D.6: Factor κ for determination of the effective length ℓ_{eff}

(10) The cylinder section j should be treated as long if:

$$\omega_j > 1{,}63 \frac{r}{t_j} \qquad \text{... (D.64)}$$

in which case the elastic critical circumferential buckling stress should be determined from:

$$\sigma_{\theta,Rcr,j} = E\left(\frac{t_j}{r}\right)^2 \left[0{,}275 + 2{,}03\left(\frac{1}{\omega_j}\cdot\frac{r}{t_j}\right)^4\right] \qquad \text{... (D.65)}$$

D.2.3.2 Buckling strength verification for circumferential compression

(1) For each cylinder section j, the conditions of 8.5 should be met, and the following check should be carried out:

$$\sigma_{\theta,Ed,j} \le \sigma_{\theta,Rcr,j} \qquad \text{... (D.66)}$$

where:

$\sigma_{\theta,Ed,j}$ is the key value of the circumferential compressive membrane stress, as detailed in the following clauses;

$\sigma_{\theta,Rcr,j}$ is the design circumferential buckling stress, as derived from the elastic critical circumferential buckling stress according to D.1.3.2.

(2) Provided that the design value of the circumferential stress resultant $n_{\theta,Ed}$ is constant throughout the length L, the key value of the circumferential compressive membrane stress in the section j, should be taken as the simple value:

$$\sigma_{\theta,Ed,j} = n_{\theta,Ed} / t_j \qquad \text{... (D.67)}$$

(3) If the design value of the circumferential stress resultant $n_{\theta,Ed}$ varies within the length L, the key value of the circumferential compressive membrane stress should be taken as a fictitious value $\sigma_{\theta,Ed,j,mod}$ determined from the maximum value of the circumferential stress resultant $n_{\theta,Ed}$ anywhere within the length L divided by the local thickness t_j (see figure D.7), determined as:

$$\sigma_{\theta,Ed,j,mod} = \max(n_{\theta,Ed}) / t_j \qquad \text{... (D.68)}$$

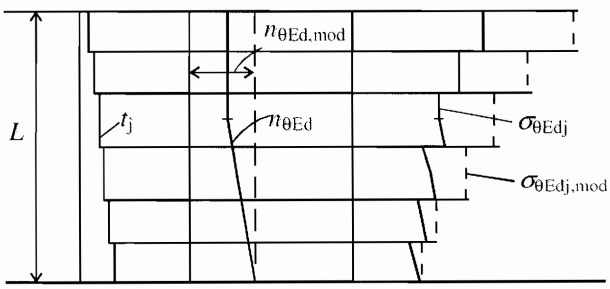


Figure D.7: Key values of the circumferential compressive membrane stress in cases where $n_{\theta,Ed}$ varies within the length L

D.2.4 Shear

D.2.4.1 Critical shear buckling stresses

(1) If no specific rule for evaluating an equivalent single cylinder of uniform wall thickness is available, the expressions of D.2.3.1 (1) to (6) may be applied.

(2) The further determination of the elastic critical shear buckling stresses may on principle be performed as in D.2.3.1 (7) to (10), but replacing the circumferential compression expressions from D.1.3.1 by the relevant shear expressions from D.1.4.1.

D.2.4.2 Buckling strength verification for shear

(1) The rules of D.2.3.2 may be applied, but replacing the circumferential compression expressions by the relevant shear expressions.

D.3 Unstiffened lap jointed cylindrical shells

D.3.1 General

D.3.1.1 Definitions

D.3.1.1.1

circumferential lap joint
A lap joint that runs in the circumferential direction around the shell axis.

D.3.1.1.2

meridional lap joint
A lap joint that runs parallel to the shell axis (meridional direction).

D.3.1.2 Geometry and stress resultants

(1) Where a cylindrical shell is constructed using lap joints (see figure D.8), the following provisions may be used in place of those set out in D.2.

(2) The following provisions apply to lap joints that increase, and those that decrease the radius of the middle surface of the shell.

(3) Where the lap joint runs in a circumferential direction around the shell axis (circumferential lap joint), the provisions of D.3.2 should be used for meridional compression.

(4) Where many lap joints run in a circumferential direction around the shell axis (circumferential lap joints) with changes of plate thickness down the shell, the provisions of D.3.3 should be used for circumferential compression.

(5) Where a continuous lap joint runs parallel to the shell axis (unstaggered meridional lap joint), the provisions of D.3.3 should be used for circumferential compression.

(6) In other cases, no special consideration need be given for the influence of lap joints on the buckling resistance.

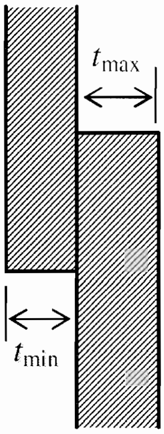


Figure D.8: Lap jointed shell

D.3.2 Meridional (axial) compression

(1) Where a lap jointed cylinder is subject to meridional compression, with circumferential lap joints, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with the design resistance reduced by the factor 0,70.

(2) Where a change of plate thickness occurs at the lap joint, the design buckling resistance may be taken as the same value as for that of the thinner plate as determined in (1).

D.3.3 Circumferential (hoop) compression

(1) Where a lap jointed cylinder is subject to circumferential compression across continuous meridional lap joints, the design buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with a reduction factor of 0,90.

(2) Where a lap jointed cylinder is subject to circumferential compression, with many circumferential lap joints and a changing plate thickness down the shell, the procedure of D.2 should be used without the geometric restrictions on joint eccentricity, and with the design buckling resistance reduced by the factor 0,90.

(3) Where the lap joints are used in both directions, with staggered placement of the meridional lap joints in alternate strakes or courses, the design buckling resistance should be evaluated as in (2), but no further resistance reduction need be applied.

D.3.4 Shear

(1) Where a lap jointed cylinder is subject to membrane shear, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, without any special allowance for the lap joints.

D.4 Unstiffened complete and truncated conical shells

D.4.1 General

D.4.1.1 Notation

In this clause the following notation is used:

h	is	the axial length (height) of the truncated cone;
L	is	the meridional length of the truncated cone ($=h/\cos\beta$);
r	is	the radius of the cone middle surface, perpendicular to axis of rotation, that varies linearly down the length;
r_1	is	the radius at the small end of the cone;
r_2	is	the radius at the large end of the cone;
β	is	the apex half angle of cone.

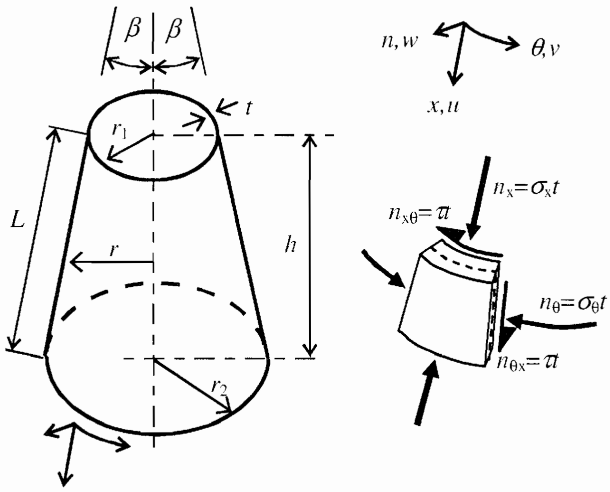


Figure D.9: Cone geometry, membrane stresses and stress resultants

D.4.1.2 Boundary conditions

(1) The following expressions should be used only for shells with boundary conditions BC 1 or BC 2 at both edges (see 5.2.2 and 8.3), with no distinction made between them. They should not be used for a shell in which any boundary condition is BC 3.

(2) The rules in this clause D.4 should be used only for the following two radial displacement restraint boundary conditions, at either end of the cone:

"cylinder condition"	$w = 0$;
"ring condition"	$u \sin\beta + w \cos\beta = 0$.

D.4.1.3 Geometry

(1) Only truncated cones of uniform wall thickness and with apex half angle $\beta \leq 65°$ (see figure D.9) are covered by the following rules.

D.4.2 Design buckling stresses

D.4.2.1 Equivalent cylinder

(1) The design buckling stresses that are needed for the buckling strength verification according to 8.5 may all be found by treating the conical shell as an equivalent cylinder of length ℓ_e and of radius r_e in which ℓ_e and r_e depend on the type of membrane stress distribution in the conical shell.

D.4.2.2 Meridional compression

(1) For cones under meridional compression, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \qquad \text{... (D.69)}$$

(2) The equivalent cylinder radius at any buckling relevant location r_e should be taken as:

$$r_e = \frac{r}{\cos\beta} \qquad \text{... (D.70)}$$

D.4.2.3 Circumferential (hoop) compression

(1) For cones under circumferential compression, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \qquad \text{... (D.71)}$$

(2) The equivalent cylinder radius r_e should be taken as:

$$r_e = \frac{(r_1 + r_2)}{2\cos\beta} \qquad \text{... (D.72)}$$

D.4.2.4 Uniform external pressure

(1) For cones under uniform external pressure q, that have either the boundary conditions BC1 at both ends or the boundary conditions BC2 at both ends, the following procedure may be used to produce a more economic design.

(2) The equivalent cylinder length ℓ_e should be taken as the lesser of:

$$\ell_e = L \qquad \text{... (D.73)}$$

and

$$\ell_e = \left(\frac{r_2}{\sin\beta}\right)(0{,}53 + 0{,}125\beta) \qquad \text{... (D.74)}$$

where the cone apex half angle β is measured in radians.

(3) For shorter cones, where the equivalent length ℓ_e is given by expression (D.73), the equivalent cylinder radius r_e should be taken as:

$$r_e = \left(\frac{0{,}55 r_1 + 0{,}45 r_2}{\cos\beta}\right) \qquad \text{... (D.75)}$$

(4) For longer cones, where the equivalent length ℓ_e is given by expression (D.74), the equivalent cylinder radius r_e should be taken as:

$$r_e = 0,71 r_2 \left[\frac{1-0,1\beta}{\cos\beta}\right] \quad \text{... (D.76)}$$

(5) The buckling strength verification should be based on the notional circumferential membrane stress:

$$\sigma_{\theta,Ed} = q\left(\frac{r_e}{t}\right) \quad \text{... (D.77)}$$

in which q is the external pressure, and no account is taken of the meridional membrane stress induced by the external pressure.

D.4.2.5 Shear

(1) For cones under membrane shear stress, the equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = h \quad \text{... (D.78)}$$

(2) The equivalent cylinder radius r_e should be taken as:

$$r_e = \left[1 + \rho_g - \frac{1}{\rho_g}\right] r_1 \cdot \cos\beta \quad \text{... (D.79)}$$

in which:

$$\rho_g = \sqrt{\frac{r_1 + r_2}{2r_1}} \quad \text{... (D.80)}$$

D.4.2.6 Uniform torsion

(1) For cones under membrane shear stress, where this is produced by uniform torsion (inducing a shear that varies linearly down the meridian), the following procedure may be used to produce a more economic design, provided $\rho_u \leq 0,8$ and the boundary conditions are BC2 at both ends.

(2) The equivalent cylinder length ℓ_e should be taken as:

$$\ell_e = L \quad \text{... (D.81)}$$

(3) The equivalent cylinder radius r_e should be taken as:

$$r_e = \left(\frac{r_1 + r_2}{2\cos\beta}\right)\left(1 - \rho_u^{2,5}\right)^{0,4} \quad \text{... (D.82)}$$

in which:

$$\rho_u = \frac{L\sin\beta}{r_2} \quad \text{... (D.83)}$$

D.4.3 Buckling strength verification

D.4.3.1 Meridional compression

(1) The buckling design check should be carried out at that point of the cone where the combination of design meridional membrane stress $\sigma_{x,Ed}$ and design meridional buckling stress $\sigma_{x,Rd}$ according to D.4.2.2 is most critical.

(2) In the case of meridional compression caused by a constant axial force on a truncated cone, both the small radius r_1 and the large radius r_2 should be considered as possible locations for the most critical position.

(3) In the case of meridional compression caused by a constant global bending moment on the cone, the small radius r_1 should be taken as the most critical.

(4) The design meridional buckling stress $\sigma_{x,Rd}$ should be determined for the equivalent cylinder according to D.1.2.

D.4.3.2 Circumferential (hoop) compression and uniform external pressure

(1) Where the circumferential compression is caused by uniform external pressure, the buckling design check should be carried out using the design circumferential membrane stress $\sigma_{\theta,Ed}$ determined using expression D.77 and the design circumferential buckling stress $\sigma_{\theta,Rd}$ according to D.4.2.1 and D.4.2.3 or D.4.2.4.

(2) Where the circumferential compression is caused by actions other than uniform external pressure, the calculated stress distribution $\sigma_{\theta,Ed}(x)$ should be replaced by a fictitious enveloping stress distribution $\sigma_{\theta,Ed,env}(x)$ that everywhere exceeds the calculated value, but which would arise from a fictitious uniform external pressure. The buckling design check should then be carried out as in paragraph (1), but using $\sigma_{\theta,Ed,env}$ instead of $\sigma_{\theta,Ed}$.

(3) The design buckling stress $\sigma_{\theta,Rd}$ should be determined for the equivalent cylinder according to D.1.3.

D.4.3.3 Shear and uniform torsion

(1) In the case of shear caused by a constant global torque on the cone, the buckling design check should be carried out using the design membrane shear stress $\tau_{x\theta,Ed}$ at the point with $r = r_e \cos\beta$ and the design buckling shear stress $\tau_{x\theta,Rd}$ according to D.4.2.1 and D.4.2.5 or D.4.2.6.

(2) Where the shear is caused by actions other than a constant global torque (such as a global shear force on the cone), the calculated stress distribution $\tau_{x\theta,Ed}(x)$ should be replaced by a fictitious enveloping stress distribution $\tau_{x\theta,Ed,env}(x)$ that everywhere exceeds the calculated value, but which would arise from a fictitious global torque. The buckling design check should then be carried out as in paragraph (1), but using $\tau_{x\theta,Ed,env}$ instead of $\tau_{x\theta,Ed}$.

(3) The design shear buckling stress $\tau_{x\theta,Rd}$ should be determined for the equivalent cylinder according to D.1.4.

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